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 $\gamma(t)$ and λ corrections at one loop**

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Published in:
Journal of High Energy Physics

DOI:
[10.1007/JHEP03\(2017\)060](https://doi.org/10.1007/JHEP03(2017)060)

Publication date:
2017

Document version
Publisher's PDF, also known as Version of record

Citation for published version (APA):
Hartmann, C., Shepherd, W., & Trott, M. (2017). The Z decay width in the SMEFT: $\gamma(t)$ and λ corrections at one loop. *Journal of High Energy Physics*, 2017(3), [60]. [https://doi.org/10.1007/JHEP03\(2017\)060](https://doi.org/10.1007/JHEP03(2017)060)

RECEIVED: December 23, 2016

REVISED: February 9, 2017

ACCEPTED: February 25, 2017

PUBLISHED: March 10, 2017

The Z decay width in the SMEFT: y_t and λ corrections at one loop

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ABSTRACT: We calculate one loop y_t and λ dependent corrections to $\bar{\Gamma}_Z$, \bar{R}_f^0 and the partial Z widths due to dimension six operators in the Standard Model Effective Field Theory (SMEFT), including finite terms. We assume CP symmetry and a $U(3)^5$ symmetry in the UV matching onto the dimension six operators, dominantly broken by the Standard Model Yukawa matrices. Corrections to these observables are predicted using the input parameters $\{\hat{\alpha}_{\text{ew}}, \hat{M}_Z, \hat{G}_F, \hat{m}_t, \hat{m}_h\}$ extracted with one loop corrections in the same limit. We show that at one loop the number of SMEFT parameters contributing to the precise LEPI pseudo-observables exceeds the number of measurements. As a result the SMEFT parameters contributing to LEP data are formally unbounded when the size of loop corrections are reached until other data is considered in a global analysis. The size of these loop effects is generically a correction of order $\sim \%$ to leading effects in the SMEFT, but we find multiple large numerical coefficients in our calculation at this order. We use a $\overline{\text{MS}}$ scheme, modified for the SMEFT, for renormalization. Some subtleties involving novel evanescent scheme dependence present in this result are explained.

KEYWORDS: Effective field theories, Beyond Standard Model

ARXIV EPRINT: [1611.09879](https://arxiv.org/abs/1611.09879)

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1 Introduction

The vast LHC data set already reported, and the more expansive data set expected to be reported during the full LHC experimental run, is unprecedented. Such a treasure trove of data on the interactions of the Standard Model (SM) fields around the electroweak scale (\bar{v}_T) enables a paradigm shift in what is possible and advisable in Effective Field Theory (EFT) studies of beyond the SM physics. With such a data set, it is possible to systematically study the SM as a real EFT. The idea is that the SM Lagrangian is just the leading order terms of a more complete theory at higher energy scales. This hypothesis is adopted by embedding the SM in an expansion including higher dimensional operators, and promoting it to the Standard Model Effective Field Theory (SMEFT).

In the SMEFT, it is assumed that $SU(2)_L \times U(1)_Y$ is spontaneously broken to $U(1)_{\text{em}}$ by the vacuum expectation value ($\langle H^\dagger H \rangle \equiv \bar{v}_T^2/2$) of the Higgs field. The minimum of the potential is determined including the effect of operators in \mathcal{L}_6 . The observed scalar is assumed to be $J^P = 0^+$ and embedded in a doublet of $SU(2)_L$, and a mass gap to the scale(s) of new physics (referred to as $\sim \Lambda$) leads to an expansion parameter $\bar{v}_T^2/\Lambda^2 < 1$. The SMEFT follows from these assumptions, and is the sum of the SM Lagrangian and a series of $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant higher dimensional operators built out of SM fields

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \dots, \quad \mathcal{L}^{(k)} = \sum_{i=1}^{n_k} \frac{C_i^{(k)}}{\Lambda^{k-4}} Q_i^{(k)} \quad \text{for } k > 4. \quad (1.1)$$

The number of non redundant operators in $\mathcal{L}^{(5)}$, $\mathcal{L}^{(6)}$, $\mathcal{L}^{(7)}$ and $\mathcal{L}^{(8)}$ is known [1–7] and a general algorithm to determine operator bases at higher orders has been established in refs. [6–9], making the SMEFT, in principle, defined to all orders in the expansion in local operators. In this work, we use a naive power counting in mass dimension so that the operators $Q_i^{(k)}$ will be suppressed by $k - 4$ powers of the cutoff scale Λ , where the $C_i^{(k)}$ are the Wilson coefficients. We generally absorb the cut off scale into the Wilson coefficients as a notational choice unless otherwise noted.

Lower energy data ($E \ll \bar{v}_T$) on flavour changing processes and dipole moments already place strong constraints on possible deviations from the SM. The SMEFT formalism is most interesting when, in spite of these constraints, deviations captured by this formalism could possibly be observed in the LHC experimental program. For this reason, it is of interest to examine the effect of higher dimensional operators in the limit that $U(3)^5$ flavour symmetry

and CP symmetry is assumed to be (at least approximately) present in a $\sim \text{TeV}$ scale physics sector. In this paper, we calculate in this limit in the SMEFT.

The measurements of the properties of the Z boson gathered at LEPI and LEP II, a subset of Electroweak Precision Data (EWPd), is important to incorporate in the SMEFT formalism. Due to the high precision and accuracy of some of these measurements, it is very important to inform expectations of possible deviations that can be measured at LHC consistent with these LEP results. For these expectations to be of most value, it is necessary to project these constraints into the SMEFT formalism consistently. This is particularly the case when drawing strong model independent conclusions, that can potentially impact the *experimental reporting of LHC data* interpreted in the SMEFT. In some recent literature very strong claims are made that certain SMEFT parameters can be set to zero in reporting LHC data, in a model independent manner, if they impact LEP measurements. This is argued to be a valid conclusion due to model independent interpretations of experimental constraints. However, the neglect of dimension eight operator effects and loop corrections involving higher dimensional operators if the cut off scale of the theory is in the $\sim \text{TeV}$ mass range is problematic if bounds on the SMEFT parameters are pushed beyond the $\sim \%$ level due to such experimental constraints. This is exactly what is required to robustly justify neglecting such effects in LHC data reporting. Naively neglecting such effects leads to conclusions that are not truly model independent, as an implicit assumption of a large value of the parameters $\Lambda/\sqrt{C_i} \gtrsim 3 \text{ TeV}$ is effectively adopted without justification by experiment. See the detailed discussion on this point in refs. [10–13]. For results and discussion (with supporting calculations) characterizing various NLO corrections in the SMEFT see refs. [13–26].

To form a more model independent and consistent picture of the constraint of LEPI measurements in the SMEFT, it is necessary to calculate the effects of higher dimensional operators at one loop on EWPd. This is the purpose of this paper. We report one loop results for the Z total width ($\bar{\Gamma}_Z$), partial widths ($\bar{\Gamma}_{Z \rightarrow \bar{\psi}\psi}$) and ratios of partial widths (\bar{R}_ψ^0) in the limit where one loop y_t, λ dependent corrections are retained.¹ Our results show that a number of SMEFT parameters are introduced at one loop that are not present at tree level in the SMEFT modification of the LEPI pseudo-observables. The number of parameters present exceeds the number of the precise LEPI pseudo-observables. As a result, the SMEFT parameters present in EWPd become formally unbounded by LEPI data alone when the typical size of one loop corrections involving these new parameters is reached. The size of this effect depends on a priori unknown Wilson coefficients, but is not robustly below the $\sim \%$ level in the SMEFT [10–13].

The prediction of $\bar{\Gamma}_Z$ in the SM, or the SMEFT, is a multi-scale problem. Measured input observables are used to specify Lagrangian parameters, which are in turn used to

¹The notation \bar{X} indicates a theoretical prediction of an observable X in the canonically normalized SMEFT. Also note that $\psi = \{\ell, u, d, \nu\}$.

predict the partial and total widths. The scales present are hierarchical²

$$0 \ll \hat{m}_\mu^2 \ll \hat{m}_Z^2 \sim \hat{m}_h^2 \sim \hat{m}_t^2. \quad (1.2)$$

Following historical conventions the input value of $\hat{\alpha}$ is run up to the scale \hat{M}_Z in this work (the running of \hat{G}_F is further suppressed). To project experimental constraints onto Wilson coefficients defined at Λ , we renormalize the theory at the scale $\mu \simeq \Lambda$. This introduces logarithmic terms in the prediction of the observables of interest, and, crucially, Wilson coefficients that are not present at tree level. A re-definition of the parameters present can be performed to absorb a subset of the logarithmic corrections at the measurement scale into effective lower scale parameters. Although such an approach is consistent with historical uses of EFT's, we do not perform such a re-definition here for the following reasons. First, these analyses aim to infer a consistent set of constraints on the parameters in the SMEFT at the matching scale [27]. Second, the matching scale cannot be far separated from the electroweak scale ($\Lambda \gg v$ as opposed to $\Lambda > v$) without introducing decoupling to a degree that these studies are not of interest. As such, non-logarithmic finite terms are not guaranteed to be significantly sub-dominant in the one loop results. In some cases, and in some schemes, the logs can dominate for classes of perturbative corrections [22]. In other cases, the logs are not very dominant numerically [14, 21, 23, 28, 29]. In either case, the logs would be reintroduced when mapping experimental constraints to the high scale theory. See refs. [13–25, 28, 29] for related discussion on NLO SMEFT results. Our results explicitly show that not all of the new parameters present in the observables, we consider, can be trivially reabsorbed into the tree level Wilson coefficients modifying EWPD, due to the multi-scale nature of the problem. The number of parameters present in LEP data at one loop in the SMEFT exceeds the number of measurements. This is the main reason the results of the (partial) explicit calculation already do not support an idea that parameters that affect LEPI data can be trivially set to zero in a truly model independent fashion in the SMEFT when reporting LHC data. This point is expected to be even further reinforced when the full one loop corrections to EWPD are known in the SMEFT.

The outline of this paper is as follows. As the technical details of this calculation contain a number of novel features, we include an extensive discussion on how the results were developed. The interplay of the counter terms for the SM fields and the pure dimension six operator counter terms that underly the cancelation of the divergences present in this calculation is discussed and demonstrated at length in sections 2, 3. Another interesting aspect of the results is the appearance of a subtle evanescent scheme dependence for dimension six operators. The cancelation of this scheme dependence is discussed and demonstrated in section B. We further develop a $\overline{\text{MS}}$ scheme for perturbation theory for SMEFT loop corrections introduced in refs. [28, 29] in this paper suitable for the SMEFT. This development includes (partial) one loop results for the input parameters $\{\hat{\alpha}, \hat{G}_F, \hat{m}_Z\}$ extracted from experimental measurements in the SMEFT in section 4. Explicit amplitude results are also reported, including finite terms, in section 3.4. We then report results for

²The notation \hat{X} indicates a measured value of an observable X in the canonically normalized SMEFT. In particular this notation is used to indicate experimentally extracted masses.

$\bar{\Gamma}_Z$, \bar{R}_f^0 and the Z partial widths in terms of these input parameters in this limit, expressed in terms of effective couplings to one loop in the SMEFT, mirroring past leading order SMEFT work, in section 6. Finally, we conclude in section 8.

2 Renormalization and notational conventions

The bare and renormalized spin one fields/couplings ($F/c = \{A, B, W, G\}/\{e, g_1, g_2, g_3\}$) are related as

$$F_\mu^{(r)} = \frac{1}{\sqrt{Z_F}} F_\mu^{(0)}, \quad c^{(r)} = \frac{1}{Z_c} \mu^{-\epsilon} c^{(0)}. \quad (2.1)$$

The factor $\mu^{-\epsilon}$ is included in the coupling relation to render the renormalized coupling dimensionless [30] and we use (0) superscripts for bare fields and (r) superscripts for renormalized quantities. In addition, the scalar field renormalization is defined as $\sqrt{Z_s} S^{(r)} = S^{(0)}$ for $S = \{h, \phi^0, \phi^\pm\}$. All the divergence subtractions are defined in the modified $\overline{\text{MS}}$ scheme for $d = 4 - 2\epsilon$ dimensions (as in refs. [28, 29]). This $\overline{\text{MS}}$ scheme utilizes the Background Field method (BFM) [31, 32] to define the divergence subtractions.³ In addition we implement on-shell renormalization conditions to define the external states wavefunctions, and a tadpole prescription to define the vev. This introduces significant technical advantages and simplifications, that primarily follow from the use of the BFM. In the BFM, the fields are split into classical and quantum components. A gauge fixing term breaks the gauge invariance of the quantum fields while maintaining the gauge invariance of the classical background fields. As a result of the BFM, for a specific choice of operator normalization, cancelations occur between insertions of the divergent renormalization factors of the SM gauge fields and couplings. This follows from the unbroken Ward identities of the classical background fields of the theory.

We use the convention that the Higgs doublet is defined as

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}i\phi^+ \\ h + \bar{v}_T + \Delta v + i\phi_0 \end{pmatrix}, \quad (2.2)$$

in R_ξ gauge (with background field gauge fixing), with ϕ^\pm and ϕ_0 the goldstone bosons. In this normalization, the physical Higgs mass in the SM is $m_h^2 = 2\lambda(\bar{v}_T + \Delta v)^2$. Finite tadpole counter terms, indicated by the introduction of Δv , are introduced to fix the one point function of the H field. This induces gauge dependent terms into the definition of the vev and Δv is formally of one loop order.⁴

We use the notation that the \bar{g}_i are the canonically normalized gauge couplings of the SM, including the effects of \mathcal{L}_6 , as defined in ref. [33]. The sign convention on the covariant derivative is $D_\mu = \partial_\mu + i g_3 A_\mu^A T^A + i g_2 W_\mu^a \sigma^a/2 + i g_1 B_\mu y_i$ with y_i the $U(1)_Y$ hypercharge generator for the state i that D acts on. Here σ_a is the Pauli matrix. The

³In this sense it is even further “modified” from some other $\overline{\text{MS}}$ conventions in the literature.

⁴Note that we have switched our notation to Δv from δv in some closely related previous works [10–12, 33, 34]. This is to consistently reserve δ notation for a correction at leading order in the tree level power counting corrections in the SMEFT (i.e. suppressed by $1/\Lambda^2$).

yukawa coupling y_t is defined with the convention that the Lagrangian mass term at leading order is $m_t = y_t \bar{v}_T / \sqrt{2}$ in the SM. ϵ_Z^μ is the polarization vector of the massive Z boson. We introduce the notation $C_Z = i \sqrt{\bar{g}_1^2 + \bar{g}_2^2} \bar{v}_T^2 / 2$ to characterize the coupling of the Z boson from SMEFT corrections. The SM weak mixing angle is defined as $s_\theta^2 = \bar{g}_1^2 / (\bar{g}_1^2 + \bar{g}_2^2)$ at leading order and Q_i is the electric charge of a state i in units of e . The SM electric charge is defined as $e = g_1 g_2 / \sqrt{g_1^2 + g_2^2}$. The chiral fermion fields are summed over the p, r, s, t flavour indices on the fields

$$U_{L/R}^p = \{u, c\}_{L/R}, \quad D_{L/R}^r = \{d, s, b\}_{L/R}, \quad \ell_{L/R}^s = \{e, \mu, \tau\}_{L/R}, \quad \nu_L^t = \{\nu_e, \nu_\mu, \nu_\tau\}. \quad (2.3)$$

In addition, the fermion indices are at times made explicit. The left handed projector is defined with the convention $P_L = (1 - \gamma_5)/2$. We discuss a subtlety involving the definition of γ_5 in d dimensions in these results in appendix B.

We use hat superscripts for measured quantities. For the measured values of the input parameters we use $\{\hat{\alpha}_{\text{ew}}, \hat{M}_Z, \hat{G}_F, \hat{m}_t, \hat{m}_h\}$. Quantities related at tree level to these input parameters are also labeled with hat superscripts. We generally use δ to indicate a correction to a SM prediction due to a power counting correction of order $1/\Lambda^2$, and Δ to indicate a correction to a SM prediction that is at least one loop in the perturbative expansion. Terms that are labeled as Δ shifts can also include power counting corrections. We retain some terms of order $1/\Lambda^4$ in the intermediate results below for ease of presentation; such higher order terms are consistently dropped in the final numerical results.

2.1 SM counter terms

For the Electroweak (EW) terms, we choose to use define the basic counter term subtractions in the modified $\overline{\text{MS}}$ scheme for the W^\pm, Z^0 fields, the gauge couplings g_1, g_2 , the vev and the scalar fields. The vev is re-normalized with the inclusion of $\bar{v}_T = \bar{v}_T^{(0)} / \sqrt{Z_v}$. We will use the subscript “div” when only the divergent part of $(\sqrt{Z_v} + \frac{\Delta v}{\bar{v}_T})_{\text{div}}$ is used. All other counter terms in the EW sector are then derived quantities at one loop. The wavefunction renormalization counter terms are only introduced for the background fields [31, 32]. One finds the relations among the SM counter terms in the BFM [35]

$$\sqrt{Z_A} Z_e = 1, \quad Z_h = Z_{\phi_\pm} = Z_{\phi_0}, \quad \sqrt{Z_B} Z_{g_1} = 1, \quad \sqrt{Z_W} Z_{g_2} = 1, \quad \sqrt{Z_G} Z_{g_3} = 1. \quad (2.4)$$

See refs. [28, 35–37] for more discussion. In the scalar sector, the BFM gives the relationship $(\sqrt{Z_v} + \frac{\Delta v}{\bar{v}_T})_{\text{div}} = \sqrt{Z_h}$ and the Higgs wavefunction renormalization is given by

$$Z_h = 1 + \frac{(3 + \xi)(\bar{g}_1^2 + 3\bar{g}_2^2)}{64\pi^2\epsilon} - \frac{N_c y_t^2}{16\pi^2\epsilon}. \quad (2.5)$$

The explicit form of the renormalization constants remaining for the SM (in our partial result) are directly determined from the diagrams in figure 1. The λ pole cancels between figure 1 b) and c) and the remaining y_t^2 pole in figure 1 a) is canceled by the vev counter term. One then identifies in the vanishing gauge coupling limit at one loop

$$Z_{g_2} = Z_W = 1. \quad (2.6)$$

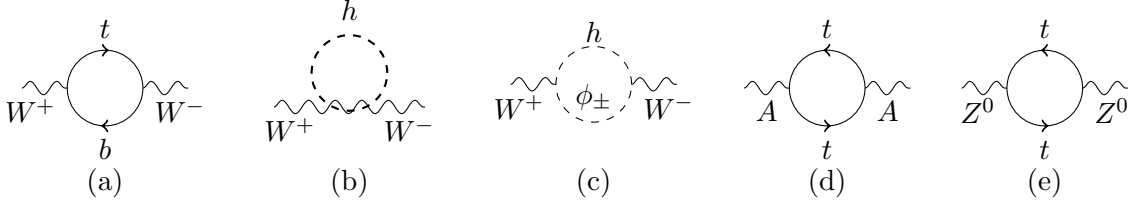


Figure 1. Subset of diagrams determining the SM counter terms for $Z_{g_1}, Z_{g_2}, Z_W, Z_{Z^0}, Z_e, Z_A$ in this calculation.

The counter term for Z_A can be directly determined from figure 1 d) and is trivially $Z_A = 1$ in the vanishing gauge coupling limit. This determines $Z_e = 1$ and hence $Z_{g_1} = 1$ from the tree level relation defining the electric coupling. Directly calculating figure 1 e) and the corresponding Z^0 diagrams for the topologies shown in b), c) determines $Z_{Z^0} = 1$ in the vanishing gauge coupling limit. Again, the λ pole cancels between the b), c) topologies for the Z^0 , and the y_t^2 pole cancels with the insertion of the vev renormalization factor.

The fermion fields are renormalized with the divergent counter term introduced as $\psi_{L/R}^{(r)} = \psi_{L/R}^{(0)} / \sqrt{Z_{L/R}^\psi}$. In this calculation, an important divergent fermion renormalisation factor is $b_L^{(r)} = b_L^{(0)} / \sqrt{Z_L^b}$. The large y_t^2 contribution to this fermion field re-normalization factor is given by

$$Z_L^b = 1 - \frac{y_t^2}{32 \pi^2 \epsilon}. \quad (2.7)$$

2.2 SMEFT counter terms

An interesting result of the complete renormalization of \mathcal{L}_{SMEFT} up to \mathcal{L}_6 [33, 38–41] is the modification of the SM counter terms, due to the introduced cut-off scale. In short, the SM with a cut-off scale — the SMEFT — is a different field theory than the SM alone, even when only considering the running of the usual SM Lagrangian parameters. A straightforward modification of the SM counter terms of this form is given for the fermion wavefunction renormalization as:

$$(Z_L^b)_{SMEFT} = 1 - \frac{y_t^2}{32 \pi^2 \epsilon} + \frac{y_t^2 \bar{v}_T^2}{16 \pi^2 \epsilon} C_{bb}^{(3)Hq}. \quad (2.8)$$

The first term in eq. (2.8) is the SM contribution, and the second term comes from the insertion of the Effective Lagrangian in the second diagram of figure 2. More subtle is the fact that the gauge couplings also run with a dependence on parameters in \mathcal{L}_6 [39]. For the $\{U(1), SU(2), SU(3)\}$ gauge fields $\{B, W, G\}$ the \mathcal{L}_4 Effective Lagrangian has the extra contribution to the SM counter terms [39]

$$\Delta Z_{g_1} = -\frac{\lambda \bar{v}_T^2}{4 \pi^2 \epsilon} g_1^2 C_{HB}, \quad \Delta Z_{g_2} = -\frac{\lambda \bar{v}_T^2}{4 \pi^2 \epsilon} g_2^2 C_{HW}, \quad \Delta Z_{g_3} = -\frac{\lambda \bar{v}_T^2}{4 \pi^2 \epsilon} g_3^2 C_{HG}. \quad (2.9)$$

In the BFM, this leads to \mathcal{L}_6 parameter dependence in the field strengths renormalization due to eq. (2.4)

$$\Delta Z_B = \frac{2 \lambda \bar{v}_T^2}{4 \pi^2 \epsilon} g_1^2 C_{HB}, \quad \Delta Z_W = \frac{2 \lambda \bar{v}_T^2}{4 \pi^2 \epsilon} g_2^2 C_{HW}, \quad \Delta Z_G = \frac{2 \lambda \bar{v}_T^2}{4 \pi^2 \epsilon} g_3^2 C_{HG}. \quad (2.10)$$

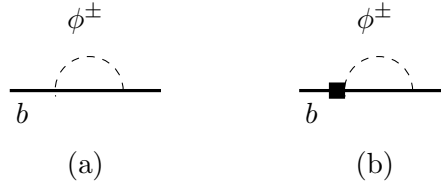


Figure 2. Fermion self energy diagrams where ϕ^\pm are the Goldstone fields. Diagram (a) is the SM contribution, while diagram (b) contains the insertion of the Effective Lagrangian including \mathcal{L}_6 corrections, indicated with a black square.

This dependence is utilized in section 4.2. In addition Z_h is also modified as

$$\Delta Z_h = \frac{m_h^2}{16\pi^2\epsilon} \left[-5C_{HD} + 14C_{H\Box} - \frac{6}{\lambda}C_H + \frac{N_c}{2\lambda}(C_{uH}y_t^2 + h.c.) - 4\frac{N_c}{\lambda}C_{Hq}^{(3)}y_t^2 \right], \quad (2.11)$$

in the BFM. This result is extracted from the modification of the running of λ and m_h^2 reported in ref. [39]. Finally, the SMEFT operator counter term matrices themselves are introduced with the convention

$$Q_i^{(0)} = Z_{i,j} Q_j^{(r)}, \quad (2.12)$$

where $Q^{(0)}$ is the bare operator, and $Q^{(r)}$ is the renormalized operator. We choose to renormalize the theory at the scale $\mu^2 = \Lambda^2$ so that the constraints derived can be directly related to the matching scale to infer properties on the underlying theory generating the SMEFT [27]. We use the Warsaw basis for the \mathcal{L}_6 Effective Lagrangian [2]. The reasons for this are extensive. First, no other construction in the literature prior to 2010 is a well defined, i.e. fully reduced by the Equations of Motion (EoM) operator basis for \mathcal{L}_6 .⁵ Second, the Warsaw basis systematically removes derivative operators, following the long established convention in EFT analyses, which directly enables recent NLO work in the SMEFT. This is not an accidental choice, but is a very well known technical advantage among EFT practitioners used in many EFT calculations. More recently, the benefit of the Warsaw basis for constructing helicity arguments for the SMEFT was also discussed in ref. [44], and then in refs. [45, 46].

Due to the presence of the mixing angles of the SM rotating the vector fields to their mass eigenstates, and the SMEFT modifications of these mixing angles, defining the large y_t and λ limit of $\bar{\Gamma}_Z$ at one loop in the SMEFT is subtle. When considering class 4, and 6 operators of the Warsaw basis, the choice to scale the Wilson coefficients of operators in \mathcal{L}_6 by gauge couplings when a field strength is present, or not, can be made. This choice can effect the terms retained in the large y_t and λ limit of interest in this paper. Our convention is to define the operator normalization as

$$\begin{aligned} Q_{uW} &= g_2 \bar{q}_{r,A} \sigma^{\mu\nu} u_s \tau_{AB}^I \tilde{H}_B W_{\mu\nu}^I, & Q_{uB} &= g_1 \bar{q}_{r,A} \sigma^{\mu\nu} u_s \tilde{H}_A B_{\mu\nu}, \\ Q_{HWB} &= g_1 g_2 H_A^\dagger \tau_{AB}^I H_B B_{\mu\nu} W_I^{\mu\nu}, & Q_{HB} &= g_1^2 H^\dagger H B_{\mu\nu} B^{\mu\nu}, \\ Q_{HW} &= g_2^2 H^\dagger H W_A^{\mu\nu} W_{\mu\nu}^A, & Q_{HG} &= g_3^2 H^\dagger H G_{\mu\nu} G^{\mu\nu}, \end{aligned} \quad (2.13)$$

⁵The Warsaw basis builds upon the results reported in refs. [1, 42, 43].

where $\tilde{H}_A = \epsilon_{AB} H^{\dagger,B}$ where $\epsilon_{12} = 1$ and $\epsilon_{AB} = -\epsilon_{BA}$. Here $prst \dots$ are flavour indices and the upper case Roman letters $AB \dots$ are $SU(2)_L$ indices. For these operators, we directly define the terms that are included below.

3 The SMEFT decay amplitude at one loop

We define the naive amplitudes leading the decay width $\bar{\Gamma}_Z = \sum_{\psi_i} \bar{\Gamma}_{Z \rightarrow \bar{\psi}_i \psi_i}$, with all fermionic final states ψ_i summed over that are kinematically allowed, schematically as

$$\mathcal{A}_{Z\bar{\psi}_i\psi_i} = \mathcal{A}_{SM} + \delta\mathcal{A}_{SMEFT} + \Delta\mathcal{A}_{SMEFT}. \quad (3.1)$$

Here the leading term \mathcal{A}_{SM} is the SM amplitude and the corresponding $\bar{\Gamma}_Z$ that follows from this expression is completely known at one loop, and partially at higher loop orders in the SM. Higher order terms that are unknown are reviewed in ref. [47] and include missing bosonic two-loop contributions and three and four loop terms of $\mathcal{O}(\alpha_{ew}^3, \alpha_{ew}^2 \alpha_s, \alpha_{ew} \alpha_s^2, \alpha_{ew} \alpha_s^3)$. The estimated size of the contributions to $\bar{\Gamma}_Z$ from these missing higher order corrections in the SM are ~ 0.5 MeV [47–49]. This is smaller *in the SM* than the experimental error quoted [50], and it deserves to be emphasized that the SM passes the hypothesis test of being consistent globally with EWPD and, in particular, the inferred value of $\hat{\Gamma}_Z$ from LEP measurements [51].

To interpret measurements of $\hat{\Gamma}_Z$ in the SMEFT, one first includes the tree level (LO) corrections to the SM predictions suppressed by $\mathcal{O}(1/\Lambda^2)$ in eq. (3.1), due to local contact operators modifying the amplitudes contributing directly to $\bar{\Gamma}_Z$. These corrections are denoted as $\delta\mathcal{A}_{SMEFT}$ and are referred to as the “naive LO contributions” in this paper. At $\mathcal{O}(1/\Lambda^2)$ further corrections are present in the predicted value of $\bar{\Gamma}_Z$ in the SMEFT, due to the mapping of the Lagrangian parameters to the measurements defining the input parameters. The full set of LO corrections due to \mathcal{L}_6 in the SMEFT are now well understood [10–12, 33, 34, 52–64]. Here we are concerned with the corrections due to \mathcal{L}_6 at one loop in this decay. We first discuss the “naive one loop amplitude” corrections of this form - denoted $\Delta\mathcal{A}_{SMEFT}$ in eq. (3.1). Subsequently the one loop improvement of measurements used to define the input parameters⁶ and interference terms of $\delta\mathcal{A}_{SMEFT}$ with the one loop contributions to the SM are discussed in section 4. We assume a narrow width approximation in an $e^+ e^- \rightarrow \bar{\psi} \psi$ process used to measure the width, to treat the Z effectively as a factorized initial state when making a theoretical prediction in the SMEFT. This approximation should be relaxed once the full one loop results are known to fully benefit from the calculations reported here.

The cancelation of the divergences in the naive amplitude is an important guide to determining the full NLO result. These cancelations occur as follows for the naive amplitudes, calculating in the broken phase of the theory.

⁶Hence the numerical values of the SMEFT Lagrangian couplings.

1 : X^3		2 : H^6		3 : $H^4 D^2$		5 : $\psi^2 H^3 + \text{h.c.}$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_H	$(H^\dagger H)^3$	$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
$Q_{\tilde{G}}$	$f^{ABC} \tilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$			Q_{HD}	$(H^\dagger D_\mu H)^* (H^\dagger D_\mu H)$	Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \tilde{H})$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$					Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$
$Q_{\tilde{W}}$	$\epsilon^{IJK} \tilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$						

4 : $X^2 H^2$		6 : $\psi^2 XH + \text{h.c.}$		7 : $\psi^2 H^2 D$	
Q_{HG}	$H^\dagger H G_{\mu\nu}^A G^{A\mu\nu}$	Q_{ew}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I H W_{\mu\nu}^I$	$Q_{Hl}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$
$Q_{H\tilde{G}}$	$H^\dagger H \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$	$Q_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{l}_p \tau^I \gamma^\mu l_r)$
Q_{HW}	$H^\dagger H W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{H} G_{\mu\nu}^A$	Q_{He}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$
$Q_{H\tilde{W}}$	$H^\dagger H \tilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{H} W_{\mu\nu}^I$	$Q_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$
Q_{HB}	$H^\dagger H B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{H} B_{\mu\nu}$	$Q_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{q}_p \tau^I \gamma^\mu q_r)$
$Q_{H\tilde{B}}$	$H^\dagger H \tilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H G_{\mu\nu}^A$	Q_{Hu}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$
Q_{HWB}	$H^\dagger \tau^I H W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I H W_{\mu\nu}^I$	Q_{Hd}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$
$Q_{H\tilde{W}B}$	$H^\dagger \tau^I H \tilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) H B_{\mu\nu}$	$Q_{Hud} + \text{h.c.}$	$i(\tilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$

8 : $(\bar{L}L)(\bar{L}L)$		8 : $(\bar{R}R)(\bar{R}R)$		8 : $(\bar{L}L)(\bar{R}R)$	
$Q_{\ell\ell}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{\ell q}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{\ell q}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$

8 : $(\bar{L}R)(\bar{R}L) + \text{h.c.}$		8 : $(\bar{L}R)(\bar{L}R) + \text{h.c.}$	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$	$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r) \epsilon_{jk} (\bar{q}_s^k d_t)$
		$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r) \epsilon_{jk} (\bar{q}_s^k T^A d_t)$
		$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r) \epsilon_{jk} (\bar{q}_s^k u_t)$
		$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r) \epsilon_{jk} (\bar{q}_s^k \sigma^{\mu\nu} u_t)$

Table 1. The independent dimension-six operators built from Standard Model fields which conserve baryon number, as given in ref. [2]. The flavour labels p, r, s, t on the Q operators are suppressed on the left hand side of the tables. This table is taken from ref. [33].

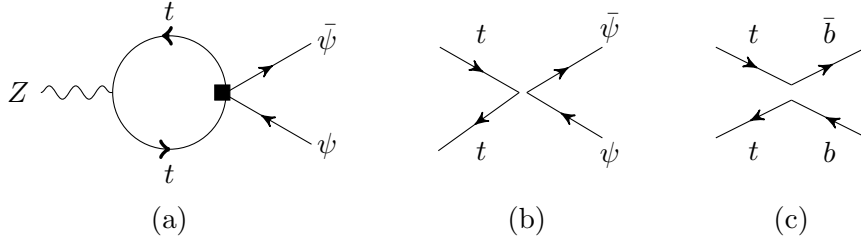


Figure 3. Direct contributions due to four fermion \mathcal{L}_6 operators to $\Delta\mathcal{A}_{SMEFT}^{C8}$. The two distinct contractions are illustrated with sub-figures (b), (c). In the figure, ψ correspond to all up, down, charged lepton and neutrino final states that are kinematically allowed.

3.1 Class 8 contributions to $\Delta\mathcal{A}_{SMEFT}$

Four fermion operators are labelled as “Class 8” in the Warsaw basis [2] used for \mathcal{L}_6 . The Class 8 operators that contribute to $\Delta\mathcal{A}_{SMEFT}$, consistent with our assumptions, are

$$O_{C8} = \{Q_{qq}^{(1)}, Q_{qq}^{(3)}, Q_{\ell q}^{(1)}, Q_{\ell q}^{(3)}, Q_{uu}, Q_{eu}, Q_{lu}, Q_{qe}, Q_{ud}^{(1)}, Q_{qu}^{(1)}, Q_{qd}^{(1)}\}. \quad (3.2)$$

The $pqrt$ flavour subscripts on each operator are suppressed. The operators are precisely defined in table 1 for completeness. The two distinct contractions in the Feynman diagrams are illustrated in figure 3. We define the notation

$$C_\epsilon = \frac{y_t^2 C_Z}{16\pi^2 \epsilon}. \quad (3.3)$$

to report results calculating in dimensional regularization with $d = 4 - 2\epsilon$ (and a universally anti-commuting γ^5). We find the divergent result in the $\overline{\text{MS}}$ scheme

$$\begin{aligned} i\Delta\mathcal{A}_\epsilon^{C8} = & -C_\epsilon N_C \bar{U}_L^p \not{\epsilon}_Z U_L^p \left[-C_{pp33}^{(1)qu} + C_{33pp}^{(1)qq} + C_{pp33}^{(1)qq} + C_{33pp}^{(3)qq} + C_{pp33}^{(3)qq} \right], \\ & -C_\epsilon N_C \bar{U}_R^p \not{\epsilon}_Z U_R^p \left[C_{33pp}^{(1)qu} - C_{33pp}^{uu} - C_{pp33}^{uu} \right], \\ & +C_\epsilon \bar{D}_L^r \not{\epsilon}_Z D_L^r \left[N_C \left(C_{rr33}^{(1)qu} - C_{33rr}^{(1)qq} - C_{rr33}^{(1)qq} + C_{33rr}^{(3)qq} + C_{rr33}^{(3)qq} \right) - 4C_{rr33}^{(3)qq} \delta_{r3} \right], \\ & +C_\epsilon N_C \bar{D}_R^r \not{\epsilon}_Z D_R^r \left[-C_{33rr}^{(1)qd} + C_{33rr}^{(1)ud} \right] - C_\epsilon N_C \bar{\nu}_L^t \not{\epsilon}_Z \nu_L^t \left[C_{tt33}^{(1)lq} + C_{tt33}^{(3)lq} - C_{tt33}^{lu} \right], \\ & -C_\epsilon N_C \bar{\ell}_L^s \not{\epsilon}_Z \ell_L^s \left[C_{ss33}^{(1)lq} - C_{ss33}^{lu} - C_{ss33}^{(3)lq} \right] - C_\epsilon N_C \bar{\ell}_R^s \not{\epsilon}_Z \ell_R^s \left[C_{33ss}^{qe} - C_{ss33}^{eu} \right]. \end{aligned} \quad (3.4)$$

The poles in eq. (3.4) are directly canceled by the insertion of the counter term matrix Z_{C7} determined in ref. [40] for the operators

$$O_{C7} = \{Q_{Hq}^{(1)pr}, Q_{Hq}^{(3)pr}, Q_{H\ell}^{(1)pr}, Q_{H\ell}^{(3)pr}, Q_{He}^{(1)pr}, Q_{Hu}^{(1)pr}, Q_{Hd}^{(1)pr}\}. \quad (3.5)$$

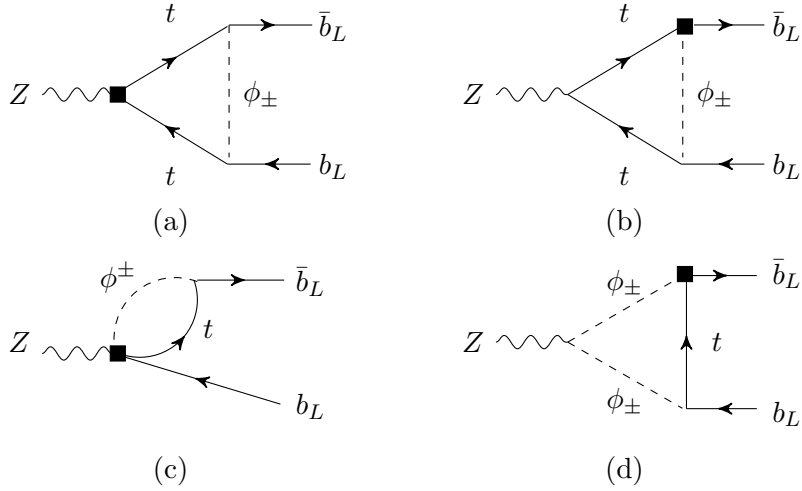


Figure 4. One loop diagrams contributing to $Z \rightarrow \bar{b}_L b_L$ decay through Class 7 operators. The insertion of the SMEFT Effective Lagrangian in the diagram is indicated with a black square. Diagrams b), c) and d) also have mirror diagrams which are not shown.

The flavour sum in this list is suppressed. The tree level expansion of the operators O_{C7} contributing to the Z decay amplitude gives

$$\begin{aligned}
 i \delta \mathcal{A}^{C7} = & C_Z \bar{U}_L^p \not{\epsilon}_Z U_L^p \left[C_{Hq}^{(1)} - C_{Hq}^{(3)} \right] + C_Z \bar{U}_R^p \not{\epsilon}_Z U_R^p \left[C_{Hu}^{pp} \right], \\
 & + C_Z \bar{D}_L^r \not{\epsilon}_Z D_L^r \left[C_{Hq}^{(1)} + C_{Hq}^{(3)} \right] + C_Z \bar{D}_R^r \not{\epsilon}_Z D_R^r \left[C_{Hd}^{rr} \right], \\
 & + C_Z \bar{\ell}_L^s \not{\epsilon}_Z \ell_L^s \left[C_{H\ell}^{(1)} + C_{H\ell}^{(3)} \right] + C_Z \bar{\ell}_R^s \not{\epsilon}_Z \ell_R^s \left[C_{He}^{ss} \right] + C_Z \bar{\nu}_L^t \not{\epsilon}_Z \nu_L^t \left[C_{H\ell}^{(1)} - C_{H\ell}^{(3)} \right].
 \end{aligned} \tag{3.6}$$

The counter term matrix for the Wilson operators expands out to cancel the poles in eq. (3.6) as $i Z_{C7} \delta \mathcal{A}^{C7} \rightarrow i \delta \mathcal{A}^{C7} - i \Delta \mathcal{A}_\epsilon^{C8}$.

3.2 Class 7 contributions to $\Delta \mathcal{A}_{SMEFT}$

A subset of Class 7 operators also contribute directly to the $Z \rightarrow \bar{b}_L b_L$ decay amplitude in the diagrams shown in figure 4. These diagrams give the result

$$i \Delta \mathcal{A}_\epsilon^{C7} = \frac{C_\epsilon}{2} \bar{b}_L \not{\epsilon}_Z b_L \left[C_{Hu}^{33} + C_{Hq}^{(3)} (6 + 4s_\theta^2 (1 - \mathcal{Q}_u)) \right]. \tag{3.7}$$

For b_L final states another contribution is present from Z_b which appears in the LSZ formula [65] as $\langle \epsilon_Z^\alpha | \bar{b}_L \gamma_\alpha b_L | \bar{b}_L b_L \rangle Z_b$. This results in a contribution to the $Z \rightarrow \bar{b}_L b_L$ matrix element

$$\langle \epsilon_Z^\alpha | \bar{b}_L \gamma_\alpha b_L | \bar{b}_L b_L \rangle Z_b = \frac{C_\epsilon}{2} \bar{b}_L \not{\epsilon}_Z b_L \left[C_{Hq}^{(3)} (1 - 4s_\theta^2 (1 - \mathcal{Q}_u)) - C_{Hq}^{(1)} \right]. \tag{3.8}$$

A final contribution for this class of operators results from the renormalization of the vev in the SMEFT. The decay amplitude from the LO insertion of the Class 7 operators is

proportional to \bar{v}_T^2 . This leads to a tadpole contribution

$$i \delta \mathcal{A}^{C7} \left(\sqrt{Z_v} + \frac{\Delta v}{\bar{v}_T} \right)_{\text{div}}^2, \quad (3.9)$$

which results in a net contribution of the form $-N_c C_\epsilon i \delta \mathcal{A}^{C7} / C_Z$. The sum of these Class 7 contributions to the matrix element of Z decay is given by

$$\langle \epsilon_Z^\alpha \bar{b}_L \gamma_\alpha b_L | \bar{b}_L b_L \rangle Z_b \left(\sqrt{Z_v} + \frac{\Delta v}{\bar{v}_T} \right)_{\text{div}}^2 = -\frac{N_c y_t^2}{16 \pi^2 \epsilon} i \delta \mathcal{A}^{C7} + \frac{C_\epsilon}{2} \bar{b}_L \not{\epsilon}_Z b_L \left[7 C_{33}^{(3)} - C_{33}^{(1)} + C_{33}^{Hu} \right]. \quad (3.10)$$

This expression directly cancels the y_t^2 dependence of the counter term matrices reported for the Class 7 operators in ref. [40].

3.3 Class 4 and 6 contributions to $\delta \mathcal{A}_{SMEFT}$ and $\Delta \mathcal{A}_{SMEFT}$

The contributions of the ‘‘Class 6’’ dipole operators to $\bar{\Gamma}_Z$ are intimately related to the contributions of the ‘‘Class 4’’ operators of ref. [2]. Due to our assumption of an approximate $U(3)^5$ symmetry in \mathcal{L}_6 , dominantly broken by the SM Yukawa matrices to satisfy flavour constraints, the flavour indices of the Class 6 operators are contracted with the SM Yukawas. This leads to flavour violation in an MFV pattern [66–71]. We neglect all fermion masses except the top quark, leaving the operators $O_6 = \{Q_{uG}, Q_{uW}, Q_{uB}\}$ to consider. For the naive amplitude in eq. (3.1), there is no contribution to $i \delta \mathcal{A}_{SMEFT}$ due to O_6 in the MFV limit considered.

The operator Q_{HWB} contributes to $\delta \mathcal{A}_{SMEFT}$ at tree level. Working in the canonically normalized SMEFT the operator \mathcal{Q}_{HWB} contributes to $\delta \mathcal{A}_{SMEFT}$ in the following manner [10, 33]

$$i \delta \mathcal{A}^{HWB} = i \sqrt{\bar{g}_1^2 + \bar{g}_2^2} \bar{\Psi}_i \not{\epsilon}_Z \Psi_i \mathcal{Q}_{\Psi_i} \left(\frac{\bar{g}_1^2 \bar{g}_2^2 (\bar{g}_2^2 - \bar{g}_1^2)}{(\bar{g}_1^2 + \bar{g}_2^2)^2} \right) \bar{v}_T^2 C_{HWB}, \quad (3.11)$$

where $\Psi_i = \{u, c, d, s, b, e, \mu, \tau\}$ and $\mathcal{Q}_{\Psi_i} = \{2/3, 2/3, -1/3, -1/3, -1/3, -1, -1, -1\}$. The counter terms are introduced into eq. (3.11) following the convention

$$i \delta \mathcal{A}^{HWB} \rightarrow i \delta \mathcal{A}^{HWB} \left(1 + \left(\sqrt{Z_v} + \frac{\Delta v}{\bar{v}_T} \right)_{\text{div}}^2 \right) \left(1 + \frac{1}{32 \pi^2 \epsilon} (2 N_c y_t^2 + 4 \lambda) \right), \quad (3.12)$$

where the relevant RGE entries of the SMEFT were reported in refs. [39, 40]. The counter term $\propto N_c y_t^2$ in this expression is exactly canceled by the vev counter term. The counter term $\propto \lambda$ in eq. (3.12) is directly canceled by the one loop contribution to the un-diagonalized kinetic term $W_{\mu\nu}^3 B^{\mu\nu}$ shown in figure 5 (a), which gives a divergent contribution

$$\frac{\bar{g}_1 \bar{g}_2 \lambda \bar{v}_T^2}{16 \pi^2 \epsilon} C_{HWB} W_{\mu\nu}^3 B^{\mu\nu}. \quad (3.13)$$

The operator Q_{HWB} also contributes directly to $\Delta \mathcal{A}_{SMEFT}$ through the diagrams shown

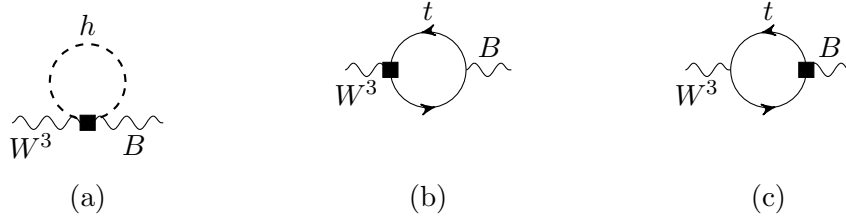


Figure 5. The contribution of Q_{HWB}, Q_{uW}, Q_{uB} to the un-diagonalized kinetic term $W^3_{\mu\nu} B^{\mu\nu}$. The insertion of the Effective Lagrangian is indicated with a black box.

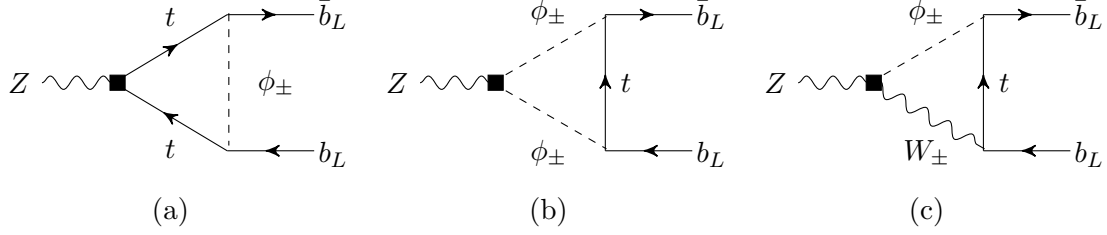


Figure 6. One loop diagrams contributing to $Z \rightarrow \bar{b}_L b_L$ decay due to Q_{HWB} . The insertion of the Effective Lagrangian in the diagram is indicated with a black square.

in figure 6. These diagrams directly give the result

$$i \Delta \mathcal{A}_\epsilon^{HWB} = C_\epsilon \bar{b}_L \not{\epsilon}_Z b_L (\mathcal{Q}_u - 1) \left[\frac{\bar{g}_1^2 \bar{g}_2^2 (\bar{g}_2^2 - \bar{g}_1^2)}{(\bar{g}_1^2 + \bar{g}_2^2)^2} \right] C_{HWB}. \quad (3.14)$$

The wavefunction renormalization of the b quarks Z_b given in eq. (2.8) cancels this ϵ pole, and is introduced as

$$i \delta \mathcal{A}^{HWB} \rightarrow i \delta \mathcal{A}^{HWB} Z_b. \quad (3.15)$$

A final term from the Class 6 operators is due to the un-diagonalized one loop two point function $\Pi_{W^3 B}$, as shown in figure 5 (b,c). The contribution of Q_{HWB} to this kinetic term at tree level is given by

$$- \frac{\bar{v}_T^2 \bar{g}_1 \bar{g}_2}{2} C_{HWB}^{(0)} W_3^{\mu\nu} B_{\mu\nu}, \quad (3.16)$$

which leads to the counter term of the Q_{HWB} operator giving the contributions

$$- \frac{\bar{v}_T^2 \bar{g}_1 \bar{g}_2}{2} W_3^{\mu\nu} B_{\mu\nu} \frac{N_c y_t^2}{32 \pi^2 \epsilon} \left[(C_{uB}^{(r)} + C_{uB}^{(r)\star}) + 2 (y_q + y_u) \left(C_{uW}^{(r)} + C_{uW}^{(r)\star} \right) \right], \quad (3.17)$$

where $y_q = 1/6$ and $y_u = 2/3$. Note that in the limit with no CP violating BSM phases in \mathcal{L}_6 : $C_{uB}^{(r)} = C_{uB}^{(r)\star}$ and $C_{uW}^{(r)} = C_{uW}^{(r)\star}$. These poles are directly canceled by the ϵ divergences in figure 5 (b,c).

3.4 Naive amplitude one loop finite amplitude

The massless limit considered in the final states of the Z decay, encourages expressing the finite term results in terms of the chiral fields of the SM, as was done in section 3.1. The

finite terms for the Class 8 operators are

$$\begin{aligned}
 i \Delta \mathcal{A}_{U,D}^{C8} = & \Delta_8^{y_t} \bar{U}_L^p \not{\epsilon}_Z U_L^p \left[C_{33pp}^{(1)qq} + C_{pp33}^{(1)qq} + C_{33pp}^{(3)qq} + C_{pp33}^{(3)qq} - C_{pp33}^{(1)qu} \right], \\
 & + \Delta_8^{y_t} \bar{U}_R^p \not{\epsilon}_Z U_R^p \left[C_{33pp}^{(1)qu} - C_{33pp}^{uu} - C_{pp33}^{uu} \right], \\
 & + \Delta_8^{y_t} \bar{D}_L^r \not{\epsilon}_Z D_L^r \left[C_{33rr}^{(1)qq} + C_{rr33}^{(1)qq} - C_{33rr}^{(3)qq} - C_{rr33}^{(3)qq} - C_{rr33}^{(1)qu} \right], \\
 & + \Delta_8^{y_t} \bar{D}_L^r \not{\epsilon}_Z D_L^r \left[\frac{4\delta_{r3}}{N_c} C_{3333}^{(3)qq} \left(-\log^{-1} \left(\frac{\Lambda^2}{\hat{m}_t^2} \right) + 1 \right) \right] + \Delta_8^{y_t} \bar{D}_R^r \not{\epsilon}_Z D_R^r \left[C_{33rr}^{(1)qd} - C_{33rr}^{(1)ud} \right], \\
 & + \Delta_8^{y_t} \bar{\ell}_L^s \not{\epsilon}_Z \ell_L^s \left[C_{ss33}^{(1)\ell q} - C_{ss33}^{(3)\ell q} - C_{ss33}^{\ell u} \right] + \Delta_8^{y_t} \bar{\ell}_R^s \not{\epsilon}_Z \ell_R^s \left[-C_{ss33}^{eu} + C_{33ss}^{qe} \right], \\
 & + \Delta_8^{y_t} \bar{\nu}_L^t \not{\epsilon}_Z \nu_L^t \left[C_{tt33}^{(1)\ell q} + C_{tt33}^{(3)\ell q} - C_{tt33}^{\ell u} \right],
 \end{aligned} \tag{3.18}$$

where

$$\Delta_8^{y_t} = -\frac{C_Z y_t^2 N_c}{16 \pi^2} \log \left[\frac{\Lambda^2}{\hat{m}_t^2} \right]. \tag{3.19}$$

\mathcal{Q}_{HWB} contributes directly to $Z \rightarrow \bar{b}_L b_L$ at one loop with the finite terms

$$i \Delta \mathcal{A}^{C4} = \frac{C_Z y_t^2}{16 \pi^2} \bar{e}^2 (\bar{c}_\theta^2 - s_\theta^2) C_{HWB} (\mathcal{Q}_u - 1) \left[\frac{3}{2} + \log \left[\frac{\Lambda^2}{\hat{m}_t^2} \right] \right] \bar{b}_L \not{\epsilon}_Z b_L, \tag{3.20}$$

and through one-loop Higgs tadpole terms as

$$i \Delta \mathcal{A}_{\Delta v}^{C4} = i \delta \mathcal{A}^{HWB} \frac{\Delta V^2}{\bar{v}_T^2}. \tag{3.21}$$

The vev shift at one loop (ΔV^2) will be defined and addressed in detail in section 4.2.⁷

The Class 7 operators contribute directly to $Z \rightarrow \bar{b}_L b_L$ at one loop with the finite terms

$$\begin{aligned}
 i \Delta \mathcal{A}^{C7} = & \frac{C_Z y_t^2}{16 \pi^2} \left[\left(-\frac{1}{4} + \frac{1}{2} \log \left[\frac{\Lambda^2}{\hat{m}_t^2} \right] \right) C_{Hu} + C_{Hq}^{(1)} \right] \bar{b}_L \not{\epsilon}_Z b_L, \\
 & + \frac{C_Z y_t^2}{16 \pi^2} C_{Hq}^{(3)} \left[\frac{1}{2} - s_\theta^2 \mathcal{Q}_b + (3 - 2\mathcal{Q}_b s_\theta^2) \log \left[\frac{\Lambda^2}{\hat{m}_t^2} \right] \right] \bar{b}_L \not{\epsilon}_Z b_L,
 \end{aligned} \tag{3.22}$$

and through Higgs tadpoles as

$$i \Delta \mathcal{A}_{\Delta v}^{C7} = i \delta \mathcal{A}^{C7} \frac{\Delta V^2}{\bar{v}_T^2}. \tag{3.23}$$

4 Input parameters, and corrections, in the SMEFT

Any prediction of $\bar{\Gamma}_Z$ in the SM or the SMEFT depends upon an input parameter set used to define the numerical values of Lagrangian parameters.⁸ The input parameter set we use

⁷Note that $\Delta V \neq \Delta v$.

⁸For discussions on input parameter choices and perturbative corrections to the input parameter set determinations in the SM, see refs. [47, 50, 72, 73].

Input parameters	Value	Ref.
$\hat{\alpha}_{\text{ew}}$	$1/137.035999139(31)$	[50, 74, 75]
\hat{m}_Z	91.1875 ± 0.0021	[50, 74–76]
\hat{G}_F	$1.1663787(6) \times 10^{-5}$	[50, 74]
\hat{m}_t	$173.21 \pm 0.51 \pm 0.71$	[50]
\hat{m}_h	$125.09 \pm 0.21 \pm 0.11$	[77]

Table 2. Current experimental best estimates of the input parameter set. We use hat superscripts to indicate when a parameter is a measured value, consistent with the notation in refs. [10–12]. We interpret the measured values of \hat{m}_t , \hat{m}_h to correspond to the $\overline{\text{MS}}$ pole mass. Quantities with units are expressed in GeV to the appropriate power.

is given in table 2. Standard Model one loop corrections to the input parameters used to define \mathcal{A}_{SM} in eq. (3.1) interfere with $\delta\mathcal{A}_{SMEFT}$. There are also one loop contributions of \mathcal{L}_6 interactions to the mapping of the input parameter set to the Lagrangian terms, which are not included in the “naive LO amplitude” defined above. These contributions can lead to shifts the same order as the interference of the $\mathcal{A}_{SM} \times \Delta\mathcal{A}_{SMEFT}$ amplitudes, for the input parameters $\{\hat{\alpha}_{\text{ew}}, \hat{m}_Z, \hat{G}_F\}$. We discuss each of these input parameters in turn in this section, characterizing the corrections that we include in our predicted value of $\bar{\Gamma}_Z$ in the SMEFT. Conversely, one loop corrections to the extraction of the input parameters \hat{m}_t and \hat{m}_h , and the mapping of these input parameters to the Lagrangian terms, are neglected, as these are two loop effects for $\bar{\Gamma}_Z$ and the related quantities reported in this paper. A class of tree level shifts to the measured pole masses in the SMEFT is unobservable in the quantities predicted, as the results are in terms of these pole masses.

4.1 $\hat{\alpha}_{\text{ew}}$ extractions

Extractions of $\hat{\alpha}_{\text{ew}}$ are dominated by $p^2 \rightarrow 0$ measurements determined by probing the Coulomb potential of a charged particle, for example in a measurement of $g - 2$ for the electron or muon. The low scale measurement extracts a different parameter in the SMEFT, compared to the SM, due to contributions to the magnetic moments of the leptons from local contact operators. These contributions are discussed in ref. [33], but are not relevant here. In the chosen $U(3)^5$ limit these corrections are proportional to light quark and lepton masses which are neglected in this work. As a result the low scale matching of α in the SMEFT proceeds in the same manner as in the SM with the mapping

$$-i \left[\frac{4\pi \hat{\alpha}(q^2)}{q^2} \right]_{q^2 \rightarrow 0} \equiv \frac{-i(\bar{e}_0 + \Delta R_{\bar{e}})^2}{q^2} \left[1 + \frac{\Sigma^{AA}(q^2)}{q^2} \right]_{q^2 \rightarrow 0} \quad (4.1)$$

In this expression, the one loop renormalization of the electric charge \bar{e} is introduced as $\bar{e} = \bar{e}_0 + \Delta R_{\bar{e}}$, where $\Delta R_{\bar{e}}$ is formally of one loop order and is fixed by renormalization conditions. The renormalization of \bar{e} in the Lagrangian is related to the two-point functions as

$$\frac{\Delta R_{\bar{e}}}{\bar{e}_0} = \frac{1}{2} \frac{\partial \Sigma_T^{AA}(p^2)}{\partial p^2} \Big|_{p^2 \rightarrow 0} \quad (4.2)$$

in the BFM, as a class of one loop corrections to $\Sigma_T^{AZ}(0)$ vanish in this case.⁹ Here $\Sigma_T^{AA}, \Sigma_T^{AZ}$ are the transverse components of the two point functions, consistent with the notation in ref. [78]. Our limit of retaining the one loop contributions proportional to λ, y_t , in the $U(3)^5$ symmetry limit, leads to a vanishing $\Delta R_{\bar{e}}$, in agreement with the results in ref. [22].

The finite terms in the low scale matching that are the largest effect are due to the vacuum polarization of the photon in the $q^2 \rightarrow 0$ limit. Following ref. [72] we rearrange this unknown term into the form

$$\left[\frac{\Sigma^{AA}(q^2)}{q^2} \right]_{q^2 \rightarrow 0} = \text{Re} \frac{\Sigma^{AA}(m_Z^2)}{m_Z^2} - \left[\frac{\text{Re} \Sigma^{AA}(m_Z^2)}{m_Z^2} - \left[\frac{\Sigma^{AA}(q^2)}{q^2} \right]_{q^2 \rightarrow 0} \right] \quad (4.3)$$

and introduce the notation

$$\nabla \alpha = \left[\frac{\text{Re} \Sigma^{AA}(m_Z^2)}{m_Z^2} - \left[\frac{\Sigma^{AA}(q^2)}{q^2} \right]_{q^2 \rightarrow 0} \right]. \quad (4.4)$$

As is standard, we decompose this quantity into perturbative and non-perturbative contributions as $\nabla \alpha = \nabla \alpha_\ell + \nabla \alpha_t + \nabla \alpha_{\text{had}} + \nabla \alpha_{\overline{\text{MS}}-\text{os}}$. The estimates for these quantities are $\nabla \alpha_\ell = 0.03150$, $\nabla \alpha_t = -0.0007$, $\nabla \alpha_{\text{had}} \approx 0.02764$ and $\nabla \alpha_{\overline{\text{MS}}-\text{os}} \approx 0.0072$ [50, 72, 79, 80] where the last term is the correction due to adjusting the value of $\nabla \alpha_{\text{had}}$ from the on-shell to $\overline{\text{MS}}$ scheme in use here. Note that we neglect any SMEFT corrections to the non-perturbative parameters, assuming them to be subdominant to other theoretical uncertainties.

4.1.1 α_{ew} running and hadronic corrections

As $\hat{\alpha}_{\text{ew}}$ is extracted in the Thompson limit ($p^2 \rightarrow 0$), the corresponding input value is run up to the scale $\mu^2 \sim m_Z^2$. It is consistent to only retain one loop contributions proportional to λ, y_t , in the $U(3)^5$ symmetry limit, neglecting this running effect in this calculation. The effect of the hadronic contributions to the vacuum polarization is due to sub-leading gauge coupling dependence that is generally neglected in this paper. However, we treat this numerically significant correction as leading order due to it being defined by significant non-perturbative effects. This nonperturbative correction to the input parameter in the SM is a numerically significant contribution leading to [50, 75]

$$1/\tilde{\alpha}_{\text{ew}}(p^2 \sim \hat{m}_Z^2) = 127.940 \pm 0.014, \quad \text{while} \quad 1/\hat{\alpha}_{\text{ew}}(p^2 \rightarrow 0) = 137.035999139(31). \quad (4.5)$$

Neglecting this modification¹⁰ would necessitate introducing a large theoretical error to this input parameter, $\mathcal{O}(10\%)$. To avoid this, we retain the (gauge coupling dependent) hadronic contributions to the vacuum polarization while neglecting the SM running. This is done by adjusting the result for $\alpha(m_Z^2)_{\text{ew}}$ by

$$\tilde{\alpha}(m_Z^2)_{\text{ew}} = \frac{\hat{\alpha}(p^2 \rightarrow 0)_{\text{ew}}}{1 - \nabla \alpha} \quad (4.6)$$

⁹The subtleties of the BFM in the SMEFT and tadpole contributions are extensive and will be discussed in more detail in a future publication.

¹⁰Our notation uses $\tilde{\alpha}_{\text{ew}}$ instead of $\hat{\alpha}_{\text{ew}}$ for $\overline{\text{MS}}$ quantities, which differs from ref. [50]. This is to avoid different uses of the tilde superscript when comparing to the LO results in refs. [10, 11].

In the remainder of this paper we use the notation with a \sim superscript to signify that this vacuum polarization correction is included. Numerically, this results in the central value $1/\tilde{\alpha}(m_Z^2)_{\text{ew}} \simeq 128.041$. This is an approximation to the full result, that would be appropriate to determine when the complete one loop of $\bar{\Gamma}_Z$ in the SMEFT is known. Numerically, this is a reasonable approximation to the result quoted in the PDG of $1/\tilde{\alpha}(m_Z^2)_{\text{ew}} = 127.940 \pm 0.014$ using higher order results in perturbation theory, in the SM.

The gauge couplings and α run differently in the SMEFT compared to the SM. These corrections are due to C_{HW} and C_{HB} dependent terms introduced in the RGEs [39]. In this calculation we retain corrections proportional to λ and y_t . If gauge coupling dependence is due to a normalization choice on the operators, we retain such effects. This is the case for this running with the normalization given in eq. (2.13). Using the results of ref. [39] as an approximation to this running,¹¹ one finds the λ dependent one loop correction to the SM value

$$\frac{1}{4\pi} \frac{\bar{g}_1^2 \bar{g}_2^2}{(\bar{g}_1^2 + \bar{g}_2^2)} [1 + \delta\alpha + \Delta\alpha] \approx 1/128.041, \quad (4.7)$$

which introduces dependence on C_{HWP} and $C_{HB} + C_{HW}$ into precise EWP measurements as

$$\delta\alpha = -\sqrt{2} \frac{4\pi \tilde{\alpha}}{\hat{G}_F} C_{HWP}^{(r)}, \quad (4.8)$$

$$\begin{aligned} \Delta\alpha &= -\sqrt{2} \frac{4\pi \tilde{\alpha}}{\hat{G}_F} C_{HWP}^{(r)} \left(\Delta V^2 + \frac{\Delta G_F}{\hat{G}_F} \right) + \frac{\tilde{\alpha}}{\pi} \hat{m}_h^2 \left(C_{HB}^{(r)} + C_{HW}^{(r)} \right) \log \left[\frac{\hat{m}_Z^2}{p^2} \right], \\ &\simeq -\sqrt{2} \frac{4\pi \tilde{\alpha}}{\hat{G}_F} C_{HWP}^{(r)} \left(\Delta V^2 + \frac{\Delta G_F}{\hat{G}_F} \right) + 0.03 \hat{m}_h^2 \left(C_{HB}^{(r)} + C_{HW}^{(r)} \right). \end{aligned} \quad (4.9)$$

If neglected, this SMEFT correction to the running can be dominant over the SM theoretical error when using the input parameter $\tilde{\alpha}(m_Z^2)_{\text{ew}}$ to define numerical values of the gauge couplings [13], depending on the *a priori* unknown value of $C_{HB} + C_{HW}$. $\Delta\alpha$ is formally counted as one loop and $1/\Lambda^2$ suppressed when constructing the result for $\bar{\Gamma}_Z$ and related quantities.

4.2 \hat{m}_Z extractions

\hat{m}_Z is extracted at LEP in a pole scan for the position of the Z resonance peak [51]. The SMEFT LO redefinition of the Lagrangian parameter that corresponds to m_Z is well known at this time [10–12, 33, 52–64, 81] and one finds [33]

$$\bar{m}_Z^2 = \frac{\bar{v}_T^2}{4} (\bar{g}_1^2 + \bar{g}_2^2) + \frac{\bar{v}_T^4}{8} C_{HD} (\bar{g}_1^2 + \bar{g}_2^2) + \frac{\bar{v}_T^4}{2} \bar{g}_1^2 \bar{g}_2^2 C_{HWP}. \quad (4.10)$$

With the \mathcal{L}_6 operators normalized as in eq. (2.13). The operators of interest when extending this result to one loop are Q_{HD} , Q_{HWP} , Q_{HW} and Q_{HB} and the relevant Effective

¹¹In the case of the SM running and the SMEFT correction, formally one should use a series of EFT's to relate the $p^2 \rightarrow 0$ limit and the higher scale $p^2 = m_Z^2$ parameter. We approximate this running without constructing the series of EFT's to relate the low and high scale. This approximation should be improved upon once the full one loop result of $\bar{\Gamma}_Z$ is known for the SMEFT. Note that we use $p^2 \simeq 0.01 \text{ GeV}^2$ for the low measurement scale value.

Lagrangian terms are

$$\begin{aligned}\mathcal{L}_{SMEFT} = & -\frac{1}{2}W_{\mu\nu}^+ W_{\mu\nu}^- - \frac{1}{4}W_{\mu\nu}^3 W_3^{\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu} - \frac{1}{4}G_{\mu\nu} G^{\mu\nu} + \frac{1}{2}\bar{v}_T^2 g_3^2 C_{HG}^{(0)} G_{\mu\nu}^A G^{A\mu\nu}, \\ & + \frac{1}{2}\bar{v}_T^2 g_2^2 C_{HW}^{(0)} W_{\mu\nu}^I W^{I\mu\nu} + \frac{1}{2}\bar{v}_T^2 g_1^2 C_{HB}^{(0)} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2}\bar{v}_T^2 C_{HWB}^{(0)} g_1 g_2 W_{\mu\nu}^3 B^{\mu\nu}, \\ & + \frac{1}{4}g_2^2 \bar{v}_T^2 W_\mu^+ W^{-\mu} + \frac{1}{8}\bar{v}_T^2 (g_2 W_\mu^3 - g_1 B_\mu)^2 + \frac{1}{16}\bar{v}_T^4 C_{HD}^{(0)} (g_2 W_\mu^3 - g_1 B_\mu)^2 + \dots \quad (4.11)\end{aligned}$$

The counter terms for the Wilson coefficients of the operators Q_{HD} , Q_{HWB} , Q_{HW} , Q_{HB} and Q_{HG} enter the calculation — denoting the renormalized operators with (r) superscripts — as

$$\begin{aligned}\mathcal{L}_6^{(0)} = & Z_{SM} Z_{i,j} C_i^{(r)} Q_j^{(r)} + \dots, \\ = & C_{HWB}^{(r)} \left(1 + \frac{2\lambda + N_c y_t^2}{16\pi^2 \epsilon}\right) Z_{SM} Q_{HWB}^{(r)} + C_{HD}^{(r)} \left(1 + \frac{6\lambda + 2N_c y_t^2}{16\pi^2 \epsilon}\right) Z_{SM} Q_{HD}^{(r)}, \\ & + \left(1 + \frac{6\lambda + N_c y_t^2}{16\pi^2 \epsilon}\right) Z_{SM} (C_{HG}^{(r)} Q_{HG}^{(r)} + C_{HB}^{(r)} Q_{HB}^{(r)} + C_{HW}^{(r)} Q_{HW}^{(r)}) + \dots \quad (4.12)\end{aligned}$$

We normalize each of the operators with the gauge coupling of the field strength(s) present in the operator as in eq. (2.13). As a result, due to the BFM, the renormalization of the gauge coupling and gauge field strengths ($F = \{W, B, G\}$) cancel for the \mathcal{L}_6 terms. The remaining Z_{SM} factors are the same for the operators Q_{HW} , Q_{HB} and Q_{HG} .

For a matrix element with no external Higgs state, the cancelation of the divergences occurs as follows. Expanding the operators about the vev results in two insertions of \bar{v}_T . The resulting y_t^2 dependent divergence cancels directly using the expression for $(\sqrt{Z_v} + \Delta v/\bar{v}_T)_{\text{div}}$. Similarly, for Q_{HD} , four insertions of \bar{v}_T leads to the y_t^2 divergence canceling directly. The cancelation of the divergent λ dependence for Q_{HWB} follows from the diagram in figure 5 as discussed in section 3.3. Also, note that the mixing with the dipole operators cancels for this operator, as shown in eq. (3.17).

The λ divergence cancels for Q_{HB} , Q_{HG} and Q_{HW} in an interesting manner. Expanding the operator Q_{HG} at one loop generates a divergence from a closed Higgs loop. A final contribution comes from expanding out the Yang-Mills field strength counter term in the SMEFT. The contributions are illustrated in figure 7. The divergence cancels as

$$\left(1 + \frac{6\lambda}{16\pi^2 \epsilon} - \frac{2\lambda}{16\pi^2 \epsilon} - \frac{4\lambda}{16\pi^2 \epsilon}\right) \frac{\bar{v}_T^2}{2} \bar{g}_3^2 G_{\mu\nu}^a G_a^{\mu\nu} C_{HG}^{(r)}. \quad (4.13)$$

A cancelation of this form also occurs for the operators Q_{HW} , Q_{HB} . The λ divergence for the operator Q_{HD} cancels through the combination of the operator counter term, and the one loop diagrams shown in figure 8. The contributions combine as

$$\left(1 + \frac{6\lambda}{16\pi^2 \epsilon} - \frac{10\lambda}{16\pi^2 \epsilon} + \frac{2\lambda}{16\pi^2 \epsilon} + \frac{4\lambda}{16\pi^2 \epsilon} - \frac{2\lambda}{16\pi^2 \epsilon}\right) \frac{\bar{v}_T^4}{16} (\bar{g}_1 B^\mu - \bar{g}_2 W_3^\mu)^2 C_{HD}^{(r)}, \quad (4.14)$$

where the order of the $1/\epsilon$ terms reflects the order of the sub-diagrams in figure 8. Using the guidance of how the divergences cancel in our chosen scheme, we can promote the tree

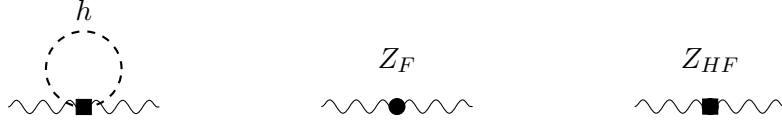


Figure 7. Cancellation of the λ divergence for the Q_{HB} , Q_{HG} and Q_{HW} operators. The effects of \mathcal{L}_6 operators and counter terms are indicated with a black square, while the insertion of the field strength counter term for the Yang-Mills \mathcal{L}_4 operator in the SMEFT is indicated with a black circle.

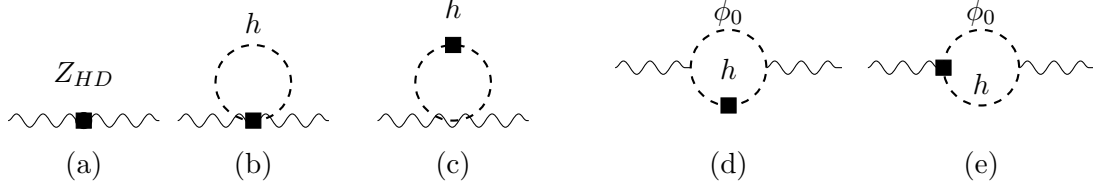


Figure 8. Cancellation of the λ divergence for the Q_{HD} operator for a matrix element with no external Higgs field. The effects of \mathcal{L}_6 operators and counter terms are indicated with a black square. Note diagram (e) has a mirror graph and corresponds to three different cases of external gauge fields.

level discussion in ref. [33] to a one loop treatment of m_Z in the SMEFT. We choose to redefine the gauge fields and couplings as

$$G_\mu^A = \mathcal{G}_\mu^A [1 + C_{HG} (\bar{v}_T^2 + \Delta V^2)], \quad \bar{g}_3 = g_3 [1 + C_{HG} (\bar{v}_T^2 + \Delta V^2)], \quad (4.15)$$

$$W_\mu^I = \mathcal{W}_\mu^I [1 + C_{HW} (\bar{v}_T^2 + \Delta V^2)], \quad \bar{g}_2 = g_2 [1 + C_{HW} (\bar{v}_T^2 + \Delta V^2)], \quad (4.16)$$

$$B_\mu = \mathcal{B}_\mu [1 + C_{HB} (\bar{v}_T^2 + \Delta V^2)], \quad \bar{g}_1 = g_1 [1 + C_{HB} (\bar{v}_T^2 + \Delta V^2)]. \quad (4.17)$$

This redefinition is performed so that $g_3 G_\mu^A = \bar{g}_3 \mathcal{G}_\mu^A$, etc. are unchanged including ΔV corrections. Here ΔV is defined as

$$\Delta V^2 = 2 \bar{v}_T \Delta v - \hat{m}_h^2 \Delta_1, \quad 16 \pi^2 \Delta_1 = 1 + \log \left(\frac{\Lambda^2}{\hat{m}_h^2} \right). \quad (4.18)$$

The electroweak terms that remain are now

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \mathcal{W}_{\mu\nu}^+ \mathcal{W}_{\mu\nu}^- - \frac{1}{4} \mathcal{W}_{\mu\nu}^3 \mathcal{W}_{\mu\nu}^3 - \frac{1}{4} \mathcal{B}_{\mu\nu} \mathcal{B}_{\mu\nu} - \frac{1}{2} \left((\bar{v}_T^2 + \Delta V^2) C_{HWB}^{(r)} + \Delta_{HWB}^{yt} \right) \bar{g}_1 \bar{g}_2 \mathcal{W}_{\mu\nu}^3 \mathcal{B}^{\mu\nu} \\ & + \left(\frac{1}{4} \bar{g}_2^2 (\bar{v}_T^2 + \Delta V^2) + \Delta m_W^2 \right) \mathcal{W}_\mu^+ \mathcal{W}^{-\mu} + \frac{1}{8} (\bar{v}_T^2 + \Delta \bar{v}_Z^2) (\bar{g}_2 \mathcal{W}_\mu^3 - \bar{g}_1 \mathcal{B}_\mu)^2, \\ & + \frac{1}{16} (\bar{v}_T^4 + \Delta \bar{v}_T^4) C_{HD}^{(r)} (\bar{g}_2 \mathcal{W}_\mu^3 - \bar{g}_1 \mathcal{B}_\mu)^2, \end{aligned} \quad (4.19)$$

with

$$\Delta m_W^2 = \frac{N_c \bar{g}_2^2}{32 \pi^2} \hat{m}_t^2 \left(\frac{1}{2} + \log \left(\frac{\Lambda^2}{\hat{m}_t^2} \right) \right) + \frac{\bar{g}_2^2}{64 \pi^2} \hat{m}_h^2 \left(\frac{3}{2} + \log \left(\frac{\Lambda^2}{\hat{m}_h^2} \right) \right), \quad (4.20)$$

$$\Delta \bar{v}_Z^2 = \Delta V^2 + \frac{N_c g_{A,t}^2}{\pi^2} \hat{m}_t^2 + \frac{\hat{m}_h^2}{8 \pi^2} \left(\frac{3}{2} + \log \left(\frac{\Lambda^2}{\hat{m}_h^2} \right) \right), \quad (4.21)$$

$$\Delta \bar{v}_T^4 = \bar{v}_T^4 \sqrt{2} \hat{G}_F \left(\frac{4}{2^{1/4}} \frac{\Delta v}{\sqrt{\hat{G}_F}} + \frac{\hat{m}_h^2}{32 \pi^2} - 3 \hat{m}_h^2 \Delta_1 \right), \quad (4.22)$$

$$\Delta_{HWB}^{yt} = -\frac{\hat{m}_t^2 N_c}{8 \pi^2} \left[C_{uB}^{(r)} + 2 (y_q + y_u) C_{uW}^{(r)} \right] \log \left[\frac{\Lambda^2}{\hat{m}_t^2} \right]. \quad (4.23)$$

On shell relations are taken in defining the p^2 dependence in the logarithms, leading to the neglect of sub-leading gauge coupling dependence. We have also neglected a class of higher dimensional operator effects in the two point functions due to massive fermion loops which require a calculation including sub-leading gauge dependence at one loop dependence to specify. The discussion in ref. [33] is then changed by promoting $\bar{v}_T^2 \rightarrow \bar{v}_T^2 + \Delta V^2$ resulting in

$$\hat{m}_Z^2 = \frac{\bar{v}_T^2}{4} (\bar{g}_1^2 + \bar{g}_2^2) + \delta m_Z^2 + \Delta m_Z^2, \quad (4.24)$$

where the leading SMEFT corrections are given by [10]

$$\frac{\delta m_Z^2}{\hat{m}_Z^2} = \frac{1}{2 \sqrt{2} \hat{G}_F} C_{HD}^{(r)} + \frac{4 \sqrt{2} \pi \tilde{\alpha}}{\hat{G}_F} C_{HWB}^{(r)}, \quad (4.25)$$

and the one loop corrections are given by

$$\Delta m_Z^2 = \left[\Delta \bar{v}_Z^2 + \frac{\Delta \bar{v}_T^4}{2} C_{HD}^{(r)} \right] \frac{(\bar{g}_1^2 + \bar{g}_2^2)}{4} + \frac{\bar{g}_1^2 \bar{g}_2^2}{2} \bar{v}_T^2 \left[(\Delta \bar{v}_Z^2 + \Delta V^2) C_{HWB}^{(r)} + \Delta_{HWB}^{yt} \right]. \quad (4.26)$$

Expressed in terms of input parameters

$$\begin{aligned} \frac{\Delta m_Z^2}{\hat{m}_Z^2} &= \sqrt{2} \Delta \bar{v}_Z^2 \hat{G}_F \left[1 - \sqrt{2} \delta G_F - \frac{\delta m_Z^2}{\hat{m}_Z^2} \right] + C_{HD}^{(r)} \left(\frac{4}{2^{1/4}} \frac{\Delta v}{\sqrt{\hat{G}_F}} + \frac{\hat{m}_h^2}{32 \pi^2} - 3 \hat{m}_h^2 \Delta_1 \right), \\ &+ 8 \pi \tilde{\alpha} \left[(\Delta \bar{v}_Z^2 + \Delta V^2) C_{HWB}^{(r)} + \Delta_{HWB}^{yt} \right]. \end{aligned} \quad (4.27)$$

Finally note ΔV^2 , \bar{g}_2^2 are expressed in terms of input parameters as

$$\Delta V^2 = \frac{2^{3/4} \Delta v}{\sqrt{\hat{G}_F}} \left(1 + \frac{\delta G_F}{\sqrt{2}} \right) - \hat{m}_h^2 \Delta_1, \quad \bar{g}_2^2 = \frac{4 \pi \tilde{\alpha}}{s_\theta^2} \left[1 + \frac{\delta s_\theta^2}{s_\theta^2} + 4 \hat{m}_W^2 C_{HWB}^{(r)} \right]. \quad (4.28)$$

The leading order SMEFT expression for \bar{g}_2^2 above was reported in ref. [10] and s_θ and δs_θ are defined in section 4.2.1. The expression for δG_F is given in section 4.3. We use these results to map a measurement of \hat{m}_Z extracted from the LEP pole scan to the Lagrangian parameters. In the experimental extraction of \hat{m}_Z radiative corrections due to SM photon emission are present and subtracted, defining this pseudo-observable. Electroweak emissions are modified in the SMEFT, but we assume that this effect is subdominant to the corrections retained. These corrections are proportional to the sub-leading gauge coupling coupling dependence and the corresponding derivative operator corrections scale as $\sim p^2/\Lambda^2$ with p^2 a scale dominated by soft emissions.

4.2.1 Δs_θ^2

The rotation angle to take the fields to mass eigenstates in the canonically normalized SMEFT is given by

$$s_\theta^2 = \frac{\bar{g}_1^2}{\bar{g}_1^2 + \bar{g}_2^2} + \frac{\bar{g}_1^2 \bar{g}_2^2 (\bar{g}_2^2 - \bar{g}_1^2)}{(\bar{g}_1^2 + \bar{g}_2^2)^2} \left((\bar{v}_T^2 + \Delta V^2) C_{HWB}^{(r)} + \Delta_{HWB}^{y_t} \right). \quad (4.29)$$

Expressing this mixing angle in terms of the input parameters is required to define the predictions of the observables of interest. The effective “measured mixing angle” is inferred in terms of the input parameters

$$s_\theta^2 = \frac{1}{2} - \frac{1}{2} \sqrt{1 - \frac{4\pi\tilde{\alpha}_{\text{ew}}}{\sqrt{2}\hat{G}_F\hat{m}_Z^2}}. \quad (4.30)$$

The short hand notation used to capture the shifts due to the SMEFT relating s_θ^2 to s_θ^2 is introduced as

$$s_\theta^2 = s_\theta^2 + \delta s_\theta^2 + \Delta s_\theta^2. \quad (4.31)$$

The leading order term is known¹² to be [10]

$$\delta s_\theta^2 = -\frac{s_\theta^2 c_\theta^2}{(1 - 2s_\theta^2)} \left[\frac{\delta m_Z^2}{\hat{m}_Z^2} + \sqrt{2} \delta G_F + 4(1 - 2s_\theta^2 c_\theta^2) \hat{m}_Z^2 C_{HWB} \right]. \quad (4.32)$$

The one loop correction is given by

$$\begin{aligned} \Delta s_\theta^2 = & -\frac{s_\theta^2 c_\theta^2}{(1 - 2s_\theta^2)} \left[\sqrt{2} \Delta G_F \left(1 - \delta\alpha + \frac{\delta m_Z^2}{\hat{m}_Z^2} \right) - \Delta\alpha + \frac{\Delta m_Z^2}{\hat{m}_Z^2} \left(1 - \delta\alpha + \sqrt{2} \delta G_F \right) \right], \\ & -s_\theta^2 c_\theta^2 4\sqrt{2} (1 - 2s_\theta^2) \hat{m}_Z^2 \hat{G}_F \left[\left(\Delta V^2 + \frac{\Delta G_F}{\hat{G}_F} \right) C_{HWB}^{(r)} + \Delta_{HWB}^{y_t} \right], \\ & -\frac{s_\theta^4 c_\theta^4}{(1 - 2s_\theta^2)} 16 \hat{m}_Z^2 \left[\frac{\Delta m_Z^2}{\hat{m}_Z^2} + \sqrt{2} \Delta G_F \right] C_{HWB}^{(r)}. \end{aligned} \quad (4.33)$$

4.3 \hat{G}_F extractions

G_F is extracted from the muon lifetime, which is dominated by the decay, $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$. The local effective interaction for muon decay is defined as the $p^2 \ll \hat{m}_W^2$ limit of the SM calculation, so that the Effective Lagrangian used is

$$\mathcal{L}_{G_F} \equiv -\frac{4G_F}{\sqrt{2}} (\bar{\nu}_\mu \gamma^\mu P_L \mu) (\bar{e} \gamma_\mu P_L \nu_e). \quad (4.34)$$

In the SMEFT, at LO the matching result [33] onto this Lagrangian is

$$-\frac{4\mathcal{G}_F}{\sqrt{2}} = -\frac{2}{\bar{v}_T^2} - 4\hat{G}_F \delta G_F. \quad (4.35)$$

¹²Note the normalization change for C_{HWB} when comparing to ref. [10].

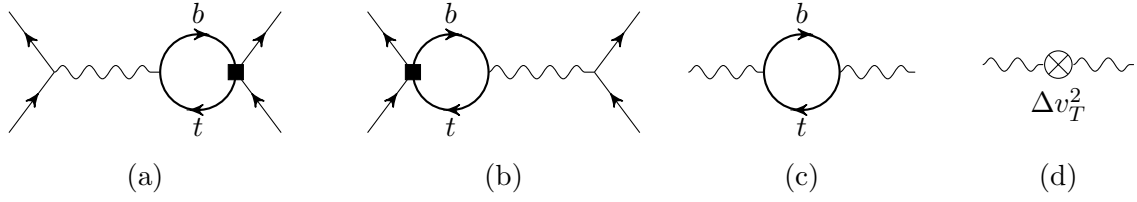


Figure 9. One loop corrections in the large y_t limit to the extraction of \hat{G}_F . Diagrams a),b) show the one loop contributions of Class 8 operators. Diagram c) corresponds to the one loop correction to the W mass in the SM in the large y_t limit. Diagram d) corresponds to the one loop correction to the vev.

where the leading order shift result is [10, 33]

$$\delta G_F = -\frac{1}{4\hat{G}_F} \left(C_{\mu e e \mu}^{ll} + C_{e \mu \mu e}^{ll} \right) + \frac{1}{2\hat{G}_F} \left(C_{ee}^{(3)} + C_{\mu\mu}^{(3)} \right). \quad (4.36)$$

The one loop correction to this result in the large λ and y_t limit is straight forward to determine in the modified $\overline{\text{MS}}$ scheme. The one loop corrections to \mathcal{L}_{G_F} in the matching result vanish when retaining only λ, y_t corrections. One loop corrections due to four fermion operators are shown in figure 9 a) and b). The finite terms for these contributions were reported in ref. [22]. The two point function of the W boson is corrected as in figure 9 c). In addition the diagrams in figure 1 b) and c) contribute to the correction to the W mass. The vev present in the W mass is corrected as $\bar{v}_T^2 \rightarrow \bar{v}_T^2 + \Delta V^2$ as illustrated in figure 9 d). The sum of these contributions lead to the result including one loop λ and y_t corrections

$$-\frac{4\mathcal{G}_F}{\sqrt{2}} = -\frac{2}{\bar{v}_T^2} \left(1 - \frac{\Delta V^2}{\bar{v}_T^2} - \frac{\Delta m_W^2}{\bar{m}_W^2} \right) - 4\hat{G}_F \delta G_F - \Delta\psi^4. \quad (4.37)$$

with a normalization change compared to ref. [10] on the coefficient of $C_{HWB}^{(r)}$. The finite terms reported in ref. [22] combined with the results derived from ref. [40] give

$$\Delta\psi^4 = \frac{N_c y_t^2}{16\pi^2} \left[-\left(C_{eett}^{(3)} + C_{\mu\mu tt}^{(3)} \right) + 2 \left(C_{eett}^{(3)} + C_{\mu\mu tt}^{(3)} - C_{ee}^{(3)} - C_{\mu\mu}^{(3)} \right) \log \left(\frac{\Lambda^2}{\hat{m}_t^2} \right) \right], \quad (4.38)$$

where the operators are renormalized and defined at the matching scale $\sim \Lambda$ to infer the limits on the high energy theory matched onto the SMEFT. We define a short hand notation to capture the one loop correction to \mathcal{G}_F in reporting results. The one loop correction ΔG_F is introduced with a normalization

$$\hat{G}_F = \frac{1}{\sqrt{2}\bar{v}_T^2} + \sqrt{2}\hat{G}_F \delta G_F + \sqrt{2}\hat{G}_F \Delta G_F, \quad (4.39)$$

where

$$\Delta G_F = -\hat{G}_F \Delta V^2 (1 - 2\sqrt{2}\delta G_F) - \frac{\Delta m_W^2}{\sqrt{2}\hat{m}_W^2} + \frac{\Delta\psi^4}{4\hat{G}_F} - \frac{\Delta m_W^2}{\sqrt{2}\hat{m}_W^2} \frac{\delta m_W^2}{\hat{m}_W^2}. \quad (4.40)$$

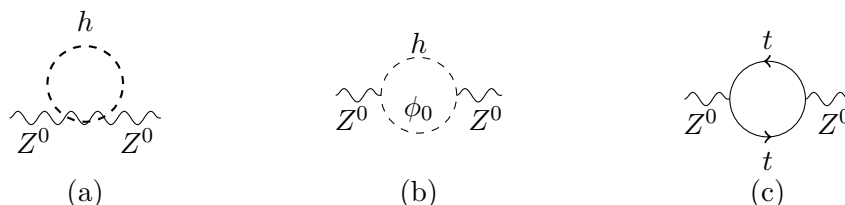


Figure 10. Diagrams determining δR_Z using on-shell renormalization conditions.

We use the definition of \hat{m}_W^2 in ref. [10] so that at tree level $\hat{m}_W^2 = c_\theta^2 \hat{m}_Z^2$ and the leading order SMEFT corrections result is

$$\delta m_W^2 = -\hat{m}_W^2 \left(\frac{\delta s_\theta^2}{s_\theta^2} + 4 \hat{m}_W^2 C_{HWB}^{(r)} + \sqrt{2} \delta G_F \right), \quad (4.41)$$

5 External state wavefunction finite terms to one loop

In addition to $\Delta \mathcal{A}_{SMEFT}$ one loop finite terms also result from the renormalization conditions adopted. As we use a modified $\overline{\text{MS}}$ scheme, these finite terms include one loop corrections to the vev (Δv), the Weinberg angle to rotate the fields to the mass eigenstate form (in $\Delta \bar{s}_\theta^2$) and the shift in the pole mass Lagrangian term $\Delta \bar{m}_Z$. These finite terms are specified in the previous sections.

It remains to define the one loop terms related to fixing the position and residue of the pole for the external states in the LSZ formula [65]. To distinguish these finite terms from those appearing in renormalization factors when an on-shell subtraction scheme is used these terms are referred to as R factors in refs. [28, 29, 82], a notation we adopt here. For $Z \rightarrow \bar{\psi} \psi$ decay these R factors appear in the corresponding S matrix element as

$$\langle Z | S | \bar{\psi}_i \psi_i \rangle = \left(1 + \frac{\Delta R_Z}{2} \right) (1 + \Delta R_{\psi_i}) i \mathcal{A}_{Z \bar{\psi}_i \psi_i}. \quad (5.1)$$

5.1 Finite terms for R_Z

The two point function of the Z boson is decomposed as

$$i \bar{\Gamma}_{\mu\nu}^Z(q) = -i g_{\mu\nu} (q^2 - \bar{m}_Z^2) - i \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \Sigma_T^Z(q^2) - i \frac{q_\mu q_\nu}{q^2} \Sigma_L^Z(q^2). \quad (5.2)$$

This leads to ΔR_Z which is defined as

$$\Delta R_Z = - \left(\text{Re} \frac{\partial \Sigma_T^Z(q^2)}{\partial q^2} \right)_{q^2 = \bar{m}_Z^2}. \quad (5.3)$$

In the limit of retaining y_t^2 and λ contributions in the vanishing gauge coupling limit $\Delta R_Z = 0$.

5.2 Finite terms for R_{ψ_i}

Following refs. [22, 23, 35, 78] the two point function for the fermion fields is defined as

$$\bar{\Gamma}_{\psi_i}(p) = i(\not{p} - m_{\psi_i}) + i\Sigma_{\psi_i}(\not{p}), \quad (5.4)$$

and in addition

$$\Sigma_{\psi_i}(\not{p}) = \left(\not{p} (P_L \Sigma_{\psi_i}^L(p^2) + P_R \Sigma_{\psi_i}^R(p^2)) + m_{\psi_i} (\Sigma_{\psi_i}^S(p^2) P_L + \Sigma_{\psi_i}^{S*}(p^2) P_R) \right), \quad (5.5)$$

and the R factors for the left-handed and right-handed fermion field fields respectively are

$$\begin{aligned} \Delta R_{\psi_i}^L &= -\tilde{\text{Re}} \Sigma_{\psi_i}^L(m_{\psi_i}^2) + \Sigma_{\psi_i}^S(m_{\psi_i}^2) - \Sigma_{\psi_i}^{S*}(m_{\psi_i}^2) \\ &\quad - m_{\psi_i}^2 \frac{\partial}{\partial p^2} \tilde{\text{Re}} \left(\Sigma_{\psi_i}^L(p^2) + \Sigma_{\psi_i}^R(p^2) + \Sigma_{\psi_i}^S(p^2) + \Sigma_{\psi_i}^{S*}(p^2) \right) \Big|_{p^2=m_{\psi_i}^2}, \\ \Delta R_{\psi_i}^R &= -\tilde{\text{Re}} \Sigma_{\psi_i}^R(m_{\psi_i}^2) \\ &\quad - m_{\psi_i}^2 \frac{\partial}{\partial p^2} \tilde{\text{Re}} \left(\Sigma_{\psi_i}^L(p^2) + \Sigma_{\psi_i}^R(p^2) + \Sigma_{\psi_i}^S(p^2) + \Sigma_{\psi_i}^{S*}(p^2) \right) \Big|_{p^2=m_{\psi_i}^2}. \end{aligned} \quad (5.6)$$

To find the fermion R factors for large y_t corrections we calculate the finite part of figure 2 for external b quarks, giving

$$\begin{aligned} \Delta R_b^L &= -\tilde{\text{Re}} \Sigma_b^L(m_b^2), \\ &= \frac{\hat{m}_t^2}{16\pi^2} \left(\sqrt{2} \hat{G}_F (1 - \sqrt{2} \delta G_F) + C_{33}^{*uH} - 2C_{Hq}^{(3)} \right) \left[-\frac{3}{4} - \frac{1}{2} \log \left[\frac{\Lambda^2}{\hat{m}_t^2} \right] \right]. \end{aligned} \quad (5.7)$$

All other fermion R factors relevant for Z decay vanish in this limit.

6 One loop results for the observables $\bar{\Gamma}_Z$, $\bar{\Gamma}_{Z \rightarrow \bar{\psi}_i \psi}$, $\bar{\Gamma}_{Z \rightarrow Had}$, \bar{R}_ℓ^0 , \bar{R}_b^0

We report partial results for $\bar{\Gamma}_{Z \rightarrow \bar{\psi}_i \psi}$ and the ratios \bar{R}_ℓ^0 , \bar{R}_b^0 in this section. It is possible to report forward-backward or other asymmetries in this limit as well, however, the neglect of gauge corrections in that case, where IR radiation and box diagram effects arising from gauge coupling corrections could significantly affect those quantities is problematic.¹³ Note that $\bar{\Gamma}_{Z \rightarrow Had} = 2\bar{\Gamma}_{Z \rightarrow \bar{u}u} + 2\bar{\Gamma}_{Z \rightarrow \bar{d}d} + \bar{\Gamma}_{Z \rightarrow \bar{b}b}$, $\bar{R}_\ell^0 = \bar{\Gamma}_{Z \rightarrow Had} / \bar{\Gamma}_{Z \rightarrow \bar{\ell}\ell}$, and $\bar{R}_b^0 = \bar{\Gamma}_{Z \rightarrow \bar{b}b} / \bar{\Gamma}_{Z \rightarrow Had}$. Results on the tree level redefinition of $\bar{\Gamma}_Z$ in the SMEFT that build up to these results, were reported in refs. [10–12, 33]. In ref. [10] δ shifts to effective vector and axial couplings are reported for the Z , for all final state fermion pairs. Working with axial and vector couplings is advantageous due to the important impact of the accidental numerical suppression of the leptonic vectorial coupling (in the SM) in studies of this form [12, 84]. Conversely, calculating the interference with SMEFT corrections in the massless final state limit can be efficiently studied using effective chiral couplings and fields. To fix our notation when using such chiral couplings and fields we note

$$\mathcal{L}_{Z,\text{eff}} = g_{Z,\text{eff}} \left(J_\mu^{Z\ell} Z^\mu + J_\mu^{Z\nu} Z^\mu + J_\mu^{Zu} Z^\mu + J_\mu^{Zd} Z^\mu \right), \quad (6.1)$$

¹³See ref. [83] for recent related discussion.

where $g_{Z,\text{eff}} = -2 \cdot 2^{1/4} \sqrt{\hat{G}_F} \hat{m}_Z$ and $(J_\mu^{Z\psi})^{pr} = \bar{\psi}_p \gamma_\mu \left[(\bar{g}_L^\psi)^{pr} P_L + (\bar{g}_R^\psi)^{pr} P_R \right] \psi_r$ for $\psi = \{u, d, \ell, \nu\}$. In a minimal linear MFV scenario $(J_\mu^{Z\psi})^{pr} \simeq (J_\mu^{Z\psi}) \delta_{pr}$ when quark mass dependence in flavour changing interactions is neglected. The coupling shifts are defined as

$$\delta(g_{L,R}^\psi)_{pr} = (\bar{g}_{L,R}^\psi)^{\text{eff}}_{pr} - (g_{L,R}^\psi)^{SM}_{pr}, \quad (6.2)$$

and

$$\delta(g_L^\psi)_{pr} = \delta(g_V^\psi)_{pr} + \delta(g_A^\psi)_{pr}, \quad \delta(g_R^\psi)_{pr} = \delta(g_V^\psi)_{pr} - \delta(g_A^\psi)_{pr}. \quad (6.3)$$

Our normalization convention is $(g_V^\psi)^{SM} = T_3/2 - \mathcal{Q}^\psi \bar{s}_\theta^2$ and $(g_A^\psi)^{SM} = T_3/2$ where $T_3 = 1/2$ for u_i, ν_i and $T_3 = -1/2$ for d_i, ℓ_i and $\mathcal{Q}^\psi = \{-1, 2/3, -1/3\}$ for $\psi = \{\ell, u, d\}$. The leading order shift results for $\delta(g_V^\psi)_{pr}$, $\delta(g_A^\psi)_{pr}$ are reported in ref. [10].¹⁴ With this convention

$$\bar{\Gamma}_{Z \rightarrow \bar{\psi}^p \psi^r} = \frac{\sqrt{2} \hat{G}_F \hat{m}_Z^3 N_c}{6 \pi} \left(\left| (\bar{g}_L^\psi)^{\text{eff}}_{pr} \right|^2 + \left| (\bar{g}_R^\psi)^{\text{eff}}_{pr} \right|^2 \right), \quad (6.4)$$

and $\bar{\Gamma}_Z = \sum_\psi \sum_{p,r} \bar{\Gamma}_{Z \rightarrow \bar{\psi}^p \psi^r}$.

6.1 One loop corrections in the SMEFT

6.1.1 Charged lepton effective couplings

For charged lepton final states the leading order (flavour symmetric) SMEFT effective coupling shifts are [10]

$$\delta(g_L^\ell)_{ss} = \delta \bar{g}_Z (g_L^\ell)_{ss}^{SM} - \frac{1}{2\sqrt{2}\hat{G}_F} \left(C_{H\ell}^{(1)}_{ss} + C_{H\ell}^{(3)}_{ss} \right) - \delta s_\theta^2, \quad (6.5)$$

$$\delta(g_R^\ell)_{ss} = \delta \bar{g}_Z (g_R^\ell)_{ss}^{SM} - \frac{1}{2\sqrt{2}\hat{G}_F} C_{He} - \delta s_\theta^2, \quad (6.6)$$

where

$$\delta \bar{g}_Z = -\frac{\delta G_F}{\sqrt{2}} - \frac{\delta m_Z^2}{2\hat{m}_Z^2} + s_\theta^2 c_\theta^2 4 \hat{m}_Z^2 C_{HWB}, \quad (6.7)$$

while the one loop corrections are

$$\begin{aligned} \Delta(g_L^\ell)_{ss} &= \Delta \bar{g}_Z (g_L^\ell)_{ss}^{SM} + \frac{N_c \hat{m}_t^2}{16 \pi^2} \log \left[\frac{\Lambda^2}{\hat{m}_t^2} \right] \left[C_{ss33}^{(1)\ell q} - C_{ss33}^{(3)\ell q} - C_{ss33}^{\ell u} \right] - \Delta s_\theta^2, \\ &\quad - \frac{1}{2} \left(\frac{\Delta G_F}{\hat{G}_F} + \Delta V^2 \right) \left(C_{H\ell}^{(1)}_{ss} + C_{H\ell}^{(3)}_{ss} \right), \end{aligned} \quad (6.8)$$

$$\begin{aligned} \Delta(g_R^\ell)_{ss} &= \Delta \bar{g}_Z (g_R^\ell)_{ss}^{SM} + \frac{N_c \hat{m}_t^2}{16 \pi^2} \log \left[\frac{\Lambda^2}{\hat{m}_t^2} \right] \left[-C_{ss33}^{eu} + C_{33ss}^{qe} \right] - \Delta s_\theta^2, \\ &\quad - \frac{1}{2} \left(\frac{\Delta G_F}{\hat{G}_F} + \Delta V^2 \right) C_{He}^{ss}, \end{aligned} \quad (6.9)$$

while

$$\begin{aligned} \Delta \bar{g}_Z &= -\frac{\Delta G_F}{\sqrt{2}} - \frac{\Delta m_Z^2}{2\hat{m}_Z^2} + \frac{\delta G_F \Delta m_Z^2}{2\sqrt{2}\hat{m}_Z^2} + \frac{\delta m_Z^2 \Delta G_F}{2\sqrt{2}\hat{m}_Z^2} + 4\hat{m}_Z^2 s_\theta^2 c_\theta^2 C_{HWB} \left[\frac{\Delta m_Z^2}{2\hat{m}_Z^2} + \frac{\Delta G_F}{\sqrt{2}} \right], \\ &\quad + \left[\left(\frac{\Delta G_F}{\hat{G}_F} + \Delta V^2 \right) C_{HWB} + \Delta_{HWB}^{yt} \right] 4\sqrt{2} s_\theta^2 c_\theta^2 \hat{m}_Z^2 \hat{G}_F. \end{aligned} \quad (6.10)$$

¹⁴Again we stress the different normalization convention for the operator Q_{HWB} when comparing to past results.

6.1.2 Neutrino effective couplings

For neutrino final states the leading order SMEFT effective coupling shifts are [10]

$$\delta(g_L^\nu)_{tt} = \delta\bar{g}_Z (g_L^\nu)_{tt}^{SM} - \frac{1}{2\sqrt{2}\hat{G}_F} \left(C_{H\ell}^{(1)} - C_{H\ell}^{(3)} \right), \quad (6.11)$$

while the one loop corrections are

$$\begin{aligned} \Delta(g_L^\nu)_{tt} = & \Delta\bar{g}_Z (g_L^\nu)_{tt}^{SM} + \frac{N_c \hat{m}_t^2}{16\pi^2} \log \left[\frac{\Lambda^2}{\hat{m}_t^2} \right] \left[C_{\ell q}^{(1)} + C_{\ell q}^{(3)} - C_{\ell u} \right]_{tt33}, \\ & - \frac{1}{2} \left(\frac{\Delta G_F}{\hat{G}_F} + \Delta V^2 \right) \left(C_{H\ell}^{(1)} - C_{H\ell}^{(3)} \right). \end{aligned} \quad (6.12)$$

6.1.3 Up quark effective couplings

For up quark final states the leading order SMEFT effective coupling shifts are [10]

$$\delta(g_L^u)_{pp} = \delta\bar{g}_Z (g_L^u)_{pp}^{SM} + \frac{2}{3}\delta s_\theta^2 + \frac{1}{2\sqrt{2}\hat{G}_F} \left(-C_{Hq}^{(1)} + C_{Hq}^{(3)} \right)_{pp}, \quad (6.13)$$

$$\delta(g_R^u)_{pp} = \delta\bar{g}_Z (g_R^u)_{pp}^{SM} + \frac{2}{3}\delta s_\theta^2 - \frac{1}{2\sqrt{2}\hat{G}_F} C_{Hu,pp}, \quad (6.14)$$

while the one loop corrections are

$$\begin{aligned} \Delta(g_L^u)_{pp} = & \Delta\bar{g}_Z (g_L^u)_{pp}^{SM} + \frac{2}{3}\Delta s_\theta^2 + \frac{1}{2} \left(\frac{\Delta G_F}{\hat{G}_F} + \Delta V^2 \right) \left(-C_{Hq}^{(1)} + C_{Hq}^{(3)} \right)_{pp}, \\ & + \frac{N_c \hat{m}_t^2}{16\pi^2} \log \left[\frac{\Lambda^2}{\hat{m}_t^2} \right] \left[C_{qq}^{(1)} + C_{qq}^{(1)} + C_{qq}^{(3)} + C_{qq}^{(3)} - C_{qu}^{(1)} \right]_{33pp}, \end{aligned} \quad (6.15)$$

$$\begin{aligned} \Delta(g_R^u)_{pp} = & \Delta\bar{g}_Z (g_R^u)_{pp}^{SM} + \frac{2}{3}\Delta s_\theta^2 + \frac{N_c \hat{m}_t^2}{16\pi^2} \log \left[\frac{\Lambda^2}{\hat{m}_t^2} \right] \left[C_{qu}^{(1)} - C_{33pp}^{uu} - C_{pp33}^{uu} \right] \\ & - \frac{1}{2} \left(\frac{\Delta G_F}{\hat{G}_F} + \Delta V^2 \right) C_{Hu,pp}. \end{aligned} \quad (6.16)$$

6.1.4 Down quark effective couplings

For down quark final states, the leading order SMEFT effective coupling shifts are [10]

$$\delta(g_L^d)_{rr} = \delta\bar{g}_Z (g_L^d)_{rr}^{SM} - \frac{1}{3}\delta s_\theta^2 - \frac{1}{2\sqrt{2}\hat{G}_F} \left(C_{Hq}^{(1)} + C_{Hq}^{(3)} \right)_{rr}, \quad (6.17)$$

$$\delta(g_R^d)_{rr} = \delta\bar{g}_Z (g_R^d)_{rr}^{SM} - \frac{1}{3}\delta s_\theta^2 - \frac{1}{2\sqrt{2}\hat{G}_F} C_{Hd,rr}, \quad (6.18)$$

while the one loop corrections are

$$\begin{aligned}
 \Delta(g_L^d)_{rr} = & \Delta\bar{g}_Z(g_L^d)_{rr}^{SM} + \frac{N_c\hat{m}_t^2}{16\pi^2} \log\left[\frac{\Lambda^2}{\hat{m}_t^2}\right] \left[C_{33rr}^{(1)} + C_{rr33}^{(1)} - C_{33rr}^{(3)} - C_{rr33}^{(3)} - C_{rr33}^{(1)} \right], \\
 & -\frac{1}{2} \left(\frac{\Delta G_F}{\hat{G}_F} + \Delta V^2 \right) \left(C_{Hq}^{(1)} + C_{Hq}^{(3)} \right) + \delta_{br} \frac{\hat{m}_t^2}{4\pi^2} \left[C_{3333}^{(3)} \left(-1 + \log\left[\frac{\Lambda^2}{\hat{m}_t^2}\right] \right) \right] - \frac{1}{3} \Delta s_\theta^2, \\
 & -\delta_{br} \frac{\hat{m}_t^2}{16\pi^2} \left[\left(\frac{1}{4} - \frac{1}{2} \log\left[\frac{\Lambda^2}{\hat{m}_t^2}\right] \right) C_{Hu} + C_{Hq}^{(1)} \right] - \delta_{br} \Delta R_b^L \left((g_L^d)_{rr}^{SM} + \delta(g_L^d)_{rr} \right), \\
 & -\delta_{br} \frac{\hat{m}_t^2}{16\pi^2} C_{Hq}^{(3)} \left[\frac{1}{2} - \mathcal{Q}_b s_\theta^2 + (3 - 2 \mathcal{Q}_b s_\theta^2) \log\left[\frac{\Lambda^2}{\hat{m}_t^2}\right] \right], \\
 & -\delta_{br} \frac{\hat{m}_t^2}{4\pi} \tilde{\alpha} (c_\theta^2 - s_\theta^2) C_{HWB} (\mathcal{Q}_u - 1) \left[\frac{3}{2} + \log\left[\frac{\Lambda^2}{\hat{m}_t^2}\right] \right],
 \end{aligned} \tag{6.19}$$

$$\begin{aligned}
 \Delta(g_R^d)_{rr} = & \Delta\bar{g}_Z(g_R^d)_{rr}^{SM} - \frac{1}{3} \Delta s_\theta^2 + \frac{N_c \hat{m}_t^2}{16\pi^2} \log\left[\frac{\Lambda^2}{\hat{m}_t^2}\right] \left[C_{33rr}^{(1)} - C_{33rr}^{(1)} \right], \\
 & -\frac{1}{2} \left(\frac{\Delta G_F}{\hat{G}_F} + \Delta V^2 \right) C_{Hd}.
 \end{aligned} \tag{6.20}$$

It is interesting to note that the left handed bottom quark coupling in the SMEFT is perturbed by a number of effects that are not present in the leptonic and up quark couplings. This observation becomes even more interesting when considering the (statistically insignificant) indication that the left handed bottom quark coupling has a greater than $\sim 2\sigma$ preference for a non-SM value in global analyses (at tree level) in the SMEFT [10–12, 85, 86]. Although we caution such tree level analyses are subject to significant theoretical uncertainties in the SMEFT [10–12], performing the one loop calculation reported in this paper provides a more solid theoretical framework that makes such a tentative indication of a deviation even more intriguing.

7 Phenomenology for $\bar{\Gamma}_Z$, $\bar{\Gamma}_{Z \rightarrow \bar{\psi}_i \psi}$, $\bar{\Gamma}_{Z \rightarrow Had}$, \bar{R}_ℓ^0 , \bar{R}_b^0

In the SMEFT, at tree level, one has [10]

$$\bar{\Gamma}(Z \rightarrow \psi \bar{\psi}) = \frac{\sqrt{2} \hat{G}_F \hat{m}_Z^3 N_c}{6\pi} \left(|\bar{g}_L^\psi|^2 + |\bar{g}_R^\psi|^2 \right), \tag{7.1}$$

$$\bar{\Gamma}(Z \rightarrow Had) = 2 \bar{\Gamma}(Z \rightarrow u \bar{u}) + 2 \bar{\Gamma}(Z \rightarrow d \bar{d}) + \bar{\Gamma}(Z \rightarrow b \bar{b}). \tag{7.2}$$

The modification of the decay widths in the SMEFT compared to the SM at leading order in the power counting (and tree level) is given as:

$$\delta \bar{\Gamma}_{Z \rightarrow \ell \bar{\ell}} = \frac{\sqrt{2} \hat{G}_F \hat{m}_Z^3}{6\pi} \left[2 g_R^\ell \delta g_R^\ell + 2 g_L^\ell \delta g_L^\ell \right] + \delta \bar{\Gamma}_{Z \rightarrow \ell \bar{\ell}, \psi^4}, \tag{7.3}$$

$$\delta \bar{\Gamma}_{Z \rightarrow \nu \bar{\nu}} = \frac{\sqrt{2} \hat{G}_F \hat{m}_Z^3}{6\pi} \left[2 g_L^\nu \delta g_L^\nu \right] + \delta \bar{\Gamma}_{Z \rightarrow \nu \bar{\nu}, \psi^4}, \tag{7.4}$$

$$\delta\bar{\Gamma}_{Z\rightarrow\text{Had}} = 2\delta\bar{\Gamma}_{Z\rightarrow\bar{u}u} + 2\delta\bar{\Gamma}_{Z\rightarrow\bar{d}d} + \delta\bar{\Gamma}_{Z\rightarrow\bar{b}b}, \quad (7.5)$$

$$= \frac{3\sqrt{2}\hat{G}_F\hat{m}_Z^3}{6\pi} \left[4g_R^u\delta g_R^u + 4g_L^u\delta g_L^u + 4g_R^d\delta g_R^d + 4g_L^d\delta g_L^d + 2g_R^b\delta g_R^b + 2g_L^b\delta g_L^b \right],$$

$$+ \delta\bar{\Gamma}_{Z\rightarrow\text{Had},\psi^4}, \quad (7.6)$$

$$\delta\bar{\Gamma}_Z = 3\delta\bar{\Gamma}_{Z\rightarrow\ell\bar{\ell}} + 3\delta\bar{\Gamma}_{Z\rightarrow\nu\bar{\nu}} + \delta\bar{\Gamma}_{\text{Had}}. \quad (7.7)$$

The shift in the ratios of decay rates follows as $R_\ell^0 = \frac{\bar{\Gamma}_{\text{Had}} + \delta\bar{\Gamma}_{\text{Had}}}{\bar{\Gamma}_{Z\rightarrow\ell\bar{\ell}} + \delta\bar{\Gamma}_{Z\rightarrow\ell\bar{\ell}}}$ and $R_b^0 = \frac{\bar{\Gamma}_{Z\rightarrow\bar{b}b} + \delta\bar{\Gamma}_{Z\rightarrow\bar{b}b}}{\bar{\Gamma}_{\text{Had}} + \delta\bar{\Gamma}_{\text{Had}}}$. These expressions directly define $\bar{R}_\psi^0 = R_\psi^0 + \delta R_\psi^0$. Note that we also indicate the dependence on interference with four fermion operators, denoted as $\delta\bar{\Gamma}_{Z\rightarrow\psi\bar{\psi},\psi^4}$ in the SMEFT reported in ref. [10]. These tree level, numerically suppressed terms should be considered when fits to LEP1 data including loop corrections are developed in more detail.

The corrections at one loop follow a similar pattern, and are given as

$$\Delta\bar{\Gamma}_{Z\rightarrow\ell\bar{\ell}} = \frac{\sqrt{2}\hat{G}_F\hat{m}_Z^3}{6\pi} \left[2g_R^\ell\Delta g_R^\ell + 2g_L^\ell\Delta g_L^\ell + 2\delta g_R^\ell\Delta g_R^\ell + 2\delta g_L^\ell\Delta g_L^\ell \right], \quad (7.8)$$

$$\Delta\bar{\Gamma}_{Z\rightarrow\nu\bar{\nu}} = \frac{\sqrt{2}\hat{G}_F\hat{m}_Z^3}{6\pi} \left[2g_L^\nu\Delta g_L^\nu + 2\delta g_L^\nu\Delta g_L^\nu \right], \quad (7.9)$$

$$\Delta\bar{\Gamma}_{Z\rightarrow\text{Had}} = 2\Delta\bar{\Gamma}_{Z\rightarrow\bar{u}u} + 2\Delta\bar{\Gamma}_{Z\rightarrow\bar{d}d} + \Delta\bar{\Gamma}_{Z\rightarrow\bar{b}b}, \quad (7.10)$$

$$= \frac{3\sqrt{2}\hat{G}_F\hat{m}_Z^3}{6\pi} \left[4(g_R^u + \delta g_R^u)\Delta g_R^u + 4(g_L^u + \delta g_L^u)\Delta g_L^u + 4(g_R^d + \delta g_R^d)\Delta g_R^d \right],$$

$$+ \frac{3\sqrt{2}\hat{G}_F\hat{m}_Z^3}{6\pi} \left[4(g_L^d + \delta g_L^d)\Delta g_L^d + 2(g_R^b + \delta g_R^b)\Delta g_R^b + 2(g_L^b + \delta g_L^b)\Delta g_L^b \right],$$

$$\Delta\bar{\Gamma}_Z = 3\Delta\bar{\Gamma}_{Z\rightarrow\ell\bar{\ell}} + 3\Delta\bar{\Gamma}_{Z\rightarrow\nu\bar{\nu}} + \Delta\bar{\Gamma}_{\text{Had}}. \quad (7.11)$$

In the cross terms of order $\delta g_{L/R}^\psi \Delta g_{L/R}^\psi$ only the terms leading order in the SMEFT power counting expansion in $\Delta g_{L/R}^\psi$ are retained in the interference term. The shift in the ratios of decay rates follows directly. In presenting numerical results, we factor the Λ dependence out of the Wilson coefficient and scale the suppression scale(s) to 1 TeV. As a result all the numerical expressions reported should be understood to be implicitly multiplied by a factor of $(1\text{ TeV})^2/\Lambda^2$.

To clarify our notational conventions, we note that we denote corrections which are linearly suppressed by a dimension-6 operator coefficient as order δ , and corrections which are present at 1-loop as order Δ . Thus, we will present results that are at order δ , corresponding to known, tree-level SMEFT effects, and new loop-level results at order $\delta\Delta$. The results are listed in the appendix.

8 Conclusions

In this article, we have calculated a set of one loop corrections to the observables $\bar{\Gamma}_Z$, $\bar{\Gamma}_{Z\rightarrow\bar{\psi}_i\psi}$, $\bar{\Gamma}_{Z\rightarrow\text{Had}}$, \bar{R}_ℓ^0 , \bar{R}_b^0 . We have developed results where λ and y_t dependent corrections are retained at one loop, while relative \bar{g}_i dependence in the loop corrections are dropped. Our results incorporate previously known Renormalization Group terms, but also include finite terms. The numerical version of the results are given in section 7. The phenomenological conclusions and implications are extensive. We postpone a detailed

discussion of some of these issues to future publications, but summarize here the most important conclusions.

- How large are the corrections? We have presented our numerical results scaled in TeV units, i.e. with the implicit multiplication of $(1\text{TeV})^2/\Lambda^2$ being understood. The relative size of the one loop SMEFT corrections to the leading order SMEFT corrections varies dramatically and is UV dependent. The results can be conservatively estimated as being a relative correction on the order of $\mathcal{O}(10\%)$ up to the ratio of the unknown Wilson coefficients. Smaller and larger corrections are also possible. This estimate does not rely on a large log enhancement and is somewhat larger than a naive expectation even so. The importance of these corrections strongly depends on the unknown Wilson coefficient matching pattern, but for this very reason, strong model independent conclusions are particularly sensitive to these effects. The logarithmic terms are not particularly dominant numerically when the cut off scale is in the TeV range.
- The most basic point is that at one loop a large number of new parameters in the SMEFT contribute, that are not present at tree level. It is not the case that a choice of the renormalization scale $\mu^2 = \hat{m}_Z^2$ can remove all of the new parameters that enter. The reason is that the SMEFT (and the SM) is a multi-scale theory and SMEFT loop corrections also result from interference with the pure SM loop corrections. At tree level, the ten SMEFT parameters that enter LEPI data in the Warsaw basis are

$$\{C_{He}, C_{Hu}, C_{Hd}, C_{H\ell}^{(1)}, C_{H\ell}^{(3)}, C_{Hq}^{(1)}, C_{Hq}^{(3)}, C_{\ell\ell}, C_{HWB}, C_{HD}\}. \quad (8.1)$$

At one loop, even in our chosen limit of $U(3)^5$ symmetry, CP symmetry, and only retaining y_t and λ corrections, the additional SMEFT parameters present in the Warsaw basis are

$$\{C_{qq}^{(1)}, C_{qq}^{(3)}, C_{qu}^{(1)}, C_{uu}, C_{qd}^{(1)}, C_{ud}^{(1)}, C_{\ell q}^{(1)}, C_{\ell q}^{(3)}, C_{\ell u}, C_{qe}, C_{eu}, C_{Hu}, C_{HB} + C_{HW}, C_{uB}, C_{uW}, C_{uH}\}.$$

With an additional sixteen parameters in our partial calculation it is clear that once the size of loop corrections are reached in the SMEFT, the parameters contributing at tree level to precise LEP data become unbounded. The size of these corrections is UV dependent, but is not robustly below the percent level [10–12], and can be greater. As a result, it is difficult to take seriously claims that LEP data constrains the parameters appearing at tree level to the per-mille level in a *completely model independent fashion*. This point has already been made in general terms in the literature but is strongly reinforced by the explicit results reported here. This once again insists that such claims do not lead to SMEFT parameters being set to zero in LHC analyses and data reporting to avoid UV bias.

- As has been noted in section 6.1, the contributions to the left handed effective bottom coupling at one loop, compared to the remaining one loop corrections to the other effective couplings, shows an interesting pattern. It is reasonable to expect the left handed effective bottom coupling to be relatively more perturbed in the SMEFT in

this limit of calculation. Whether this points to an underlying explanation of some partially decoupled new physics effects with important one loop corrections in the SMEFT leading to the (statistically insignificant) anomaly in LEPI data in the left handed bottom coupling is inconclusive, but very interesting to consider.

- We have reported numerical results in section 7 making the tadpole finite term $\Delta\bar{v}_T$ explicit. The purpose of presenting our results in this manner is to emphasize that the tadpole contributions do affect the result in our $\overline{\text{MS}}$ scheme. The naive expectation is that a pure on-shell renormalization scheme will lead to a cancelation of tadpole effects. This expectation is based on the assumption that the manner in which the on-shell scheme will fix Lagrangian parameters in the SM, will carry over to the SMEFT. There is ample reason to question this naive expectation. One way to understand this is to consider the “evanescent” scheme dependence (see appendix B) present at one loop until one considers experimentally extracting the Wilson coefficients, to then use such measured quantities to predict a deviation in a related process. This implies that a systematic set of renormalization conditions is required in the SMEFT to unambiguously fix all parameters in the Lagrangian including the SMEFT Wilson coefficients, beyond those usually employed in an on-shell scheme for the SM. Clearly a full one loop calculation, including gauge coupling dependence is required to conclusively examine the tadpole issue in the SMEFT. Related to this point, we stress that we make no particular claim that the modified $\overline{\text{MS}}$ scheme we use is in any way preferred, this is simply a subtraction scheme choice with its own benefits and challenges. Further, the demonstration of the robustness of interpreting W mass measurements in the SMEFT [34] supports the on-shell scheme employed and developed for the SMEFT in refs. [22, 23] being of at least equal interest.
- We have presented results with the theory renormalized at the scale $\mu^2 = \Lambda^2$ in order to make it more transparent to infer constraints on underlying models integrated out and matched onto the SMEFT. To form model independent conclusions it is necessary to seriously engage with these loop corrections and consider the further loop effects that are still uncalculated, when interpreting strong LEP experimental constraints. In much recent literature, SMEFT loop corrections have generally been (implicitly) assumed to be zero and ignored. Alternatively, some works have argued these effects are not important by essentially invoking UV bias and assumptions. These approaches do not lead to model independent interpretations of precise LEP data. We encourage the reader to use these results to decide for themselves the size of these corrections in various models of interest, and what is a reasonable model independent degree of constraint to assert due to LEP data in the SMEFT.¹⁵

Our results are positive for the hopes of physics beyond the SM being discoverable in the long term LHC experimental program, despite LEP constraints. We also believe our results strongly encourage the further development of the SMEFT as a meaningful paradigm for interpreting precise experimental measurements. Loop corrections can be

¹⁵The results of this calculation can also be compared to arguments and claims in YR4 [87] and ref. [13].

directly calculated when treating the SMEFT as a well defined field theory. This reduces significant theoretical errors and avoids implicit UV bias in naive leading order analyses. Such calculations are simply required for a precise and serious interpretation of the most precise experimental measurements considering the global data set. Fortunately, these calculations can be directly performed using standard EFT methods. There is still an enormous amount of work to do to project the data consistently into the SMEFT.

Acknowledgments

MT acknowledges generous support from the Villum Fonden and the Discovery center. WS acknowledges generous support from the Danish Council for Independent Research and the Alexander von Humboldt Foundation, in the framework of the Sofja Kovalevskaja Award 2016, endowed by the German Federal Ministry of Education and Research. MT and CH thank Joaquin Blasco for discussions in a very early stage of this work. MT also thanks Ilaria Brivio, Aneesh Manohar, Giampiero Passarino and Benjamin Pecjak for discussions and (in a sense) the Higgs XS working group for a motivating environment to develop these results. We also thank Ilaria Brivio and Aneesh Manohar for comments on the manuscript.

A One loop vev finite terms

In the λ, y_t dominance limit of interest in this work, the tadpole correction to the one point function of the Higgs field has the form [28, 29]

$$\frac{\Delta \bar{v}_T}{\bar{v}_T} = \frac{3\lambda}{16\pi^2} \left(1 + \log \left[\frac{\mu^2}{m_h^2} \right] \right) - \frac{2N_c}{16\pi^2} \frac{m_t^2}{m_h^2} y_t^2 \left(1 + \log \left[\frac{\mu^2}{m_t^2} \right] \right) + \mathcal{O} \left(\frac{\bar{v}_T^2}{16\pi^2 \Lambda^2} \right) + \dots \quad (\text{A.1})$$

Additional corrections are also present in the one point function of the h field in the SMEFT, indicated here as $\mathcal{O} \left(\frac{\bar{v}_T^2}{16\pi^2 \Lambda^2} \right)$, proportional to $1/\Lambda^2$ at one loop. We postpone a detailed discussion of these terms to a future publication.

B Class 8 scheme dependence in d dimensions

A subtlety is present in defining the one loop finite terms in the SMEFT for Class 8 operators, when calculating $\bar{\Gamma}_Z$. As is well known, see for example related discussion in refs. [88–93], $d \neq 4$ scheme dependence can exist in a calculation of a particular observable at one loop, when considering finite terms in amplitudes. This occurs in the case of interest here, due to an odd parity fermion loop in d dimensions due to a number of Class 8 operators. We refer to this scheme dependence as “evanescent”, due to its relationship to the issues of d dimensional effects and evanescent operators discussed in refs. [88, 89].

Various schemes can be chosen to define γ_5 in d dimensions. Here we consider Naive dimensional regularization, which assumes that γ_5 is anti-commuting with the other γ matrices; and the t’Hooft- Veltman scheme (HV) [94], in which γ_5 is anti-commuting with γ matrices in $d = 4$ dimensions and commuting in $d = -2\epsilon$ dimensions. Dimensional reduction, in which $d = 4$ is assumed for γ_5 , is another possibility. We use naive dimensional regularization in our main results including finite terms. This requires some clarification of the

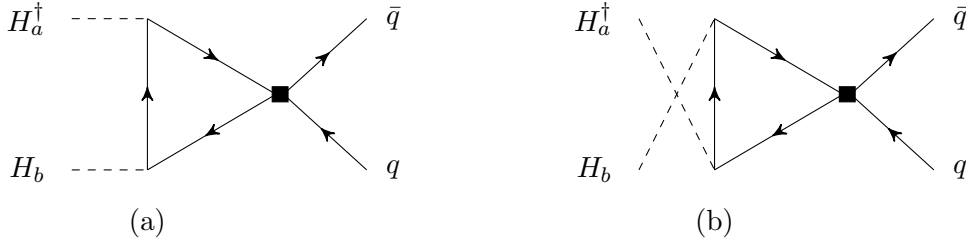


Figure 11. Evanescent one loop matching correction onto $C_{Hq}^{(1)}$.

cancellation of this $d \neq 4$ “evanescent” scheme dependence when considering relations between measurable quantities, which in this case formally includes the renormalized SMEFT Wilson coefficients. If these Wilson coefficients are non-zero, the extraction of these parameters corresponds to another set of input parameters in general. The expectation based on known examples is that the scheme dependence is unphysical, and will cancel in physical relations between measured quantities [88, 89]. We find that this is indeed the case as follows.

Consider the contribution of the operator $\mathcal{Q}_{qq}^{(1)}$ to the naive amplitude for the decay $Z \rightarrow \bar{\psi}_p \psi_p$. The finite terms proportional to y_t^2 in the HV scheme are found to be

$$i\mathcal{A}^{HV} = -\frac{i\delta_{pr}}{16\pi^2} \left(C_{33pr}^{(1)} + C_{pr33}^{(1)} \right) m_z v y_t^2 \bar{u}_p \tilde{\gamma}_\alpha P_L u_r, \quad (\text{B.1})$$

for the $p = r$ flavour indices summed over in this decay. Here the \sim superscript indicates an explicitly 4 dimensional γ matrix. This result differs from the result obtained in a naive dimensional regularization calculation by a factor

$$i\mathcal{A}^{HV-NDR} = -\frac{i\delta_{pr}}{16\pi^2} \left(C_{33pr}^{(1)} + C_{pr33}^{(1)} \right) m_z v y_t^2 \bar{u}_p \tilde{\gamma}_\alpha P_L u_r. \quad (\text{B.2})$$

For the two schemes to lead to the same result, we expect the above extra contribution in the HV scheme to be canceled by a finite matching correction at one loop. This occurs for the Wilson coefficient $C_{Hq}^{(1)}$ contributing to the decay, due to an evanescent $d \neq 4$ effect. Calculating the two diagrams shown in figure 11 in the HV scheme with $d \neq 4$ in the unbroken phase of the theory, one finds the required finite matching correction

$$C_{Hq}^{(1)} = C_{Hq}^{(1)} + \frac{1}{48\pi^2} \left(C_{prst}^{(1)} + C_{stpr}^{(1)} \right) (2[Y_u^\dagger Y_u]^{st} + [Y_u Y_u^\dagger]^{st}). \quad (\text{B.3})$$

This exactly cancels the scheme dependence in eq. (B.2), when inserted in the tree level matrix element for $C_{Hq}^{(1)}$. In this manner, the evanescent scheme dependence for the Class 8 operators cancels out of relations between observables, as expected.

This result is equivalent to that achieved by the usual method of introducing explicit counter terms to regain gauge invariance, but makes clear that this class of scheme choices is no different from any other. Ultimately, the consistent application of a renormalization scheme to both input parameters and predicted observables will cancel the scheme choices made.

C Numerical expressions for Z-pole observables

C.1 Numerical results for $\bar{\Gamma}_{Z \rightarrow \bar{\ell}\ell}$

The δ correction to $\bar{\Gamma}_{Z \rightarrow \bar{\ell}\ell}$ is given by

$$\frac{\delta \bar{\Gamma}_{Z \rightarrow \bar{\ell}\ell}}{10^{-2}} = \left[1.07 C_{H\ell}^{(1)} - 0.939 C_{He} - 0.298 C_{HD} - 0.117 C_{H\ell}^{(3)} - 0.049 C_{HWB} + 0.595 C_{\ell\ell} \right]. \quad (\text{C.1})$$

The $\delta \Delta$ correction to $\bar{\Gamma}_{Z \rightarrow \bar{\ell}\ell}$ has the contributions

$$\begin{aligned} \frac{\delta \Delta \bar{\Gamma}_{Z \rightarrow \bar{\ell}\ell}}{10^{-3}} = & \left[(0.071 \Delta \bar{v}_T + 0.201) C_{He} + (0.153 \Delta \bar{v}_T + 0.065) C_{H\ell}^{(1)} - (0.094 \Delta \bar{v}_T + 0.118) C_{HD}, \right. \\ & - (0.390 \Delta \bar{v}_T + 0.476) C_{H\ell}^{(3)} - (0.085 \Delta \bar{v}_T + 0.117) C_{HWB} + 0.271 (\Delta \bar{v}_T + 1) C_{\ell\ell}, \\ & \left. + 0.112 C_{\ell q}^{(3)} + 0.007 (C_{HB} + C_{HW}) + 0.797 \Delta \bar{v}_T \right], \end{aligned} \quad (\text{C.2})$$

and $(\delta \Delta \bar{\Gamma}_{Z \rightarrow \bar{\ell}\ell} / 10^{-3}) \times (\Lambda^2 / (1 \text{ TeV})^2)$ also has the logarithmic terms

$$\begin{aligned} \frac{\delta \Delta \bar{\Gamma}_{Z \rightarrow \bar{\ell}\ell}}{10^{-3}} = & \left[0.342 C_{He} + 0.153 C_{H\ell}^{(1)} - 0.218 C_{HD} - 0.657 C_{H\ell}^{(3)} - 0.213 C_{HWB} + 0.517 C_{\ell\ell}, \right. \\ & - 0.022 C_{\ell q}^{(3)} + 0.202 (C_{\ell u} - C_{\ell q}^{(1)}) + 0.176 C_{qe} - 0.027 C_{uW} - 0.016 C_{uB} \left. \right] \log \left[\frac{\Lambda^2}{\hat{m}_t^2} \right] \\ & + \left[6.21 \times 10^{-6} C_{HD} + 0.015 C_{He} - 0.018 C_{H\ell}^{(1)} + 0.002 C_{H\ell}^{(3)} \right. \\ & \left. - (1.39 C_{HWB} + 9.73 C_{\ell\ell}) 10^{-3} \right] \log \left[\frac{\Lambda^2}{\hat{m}_h^2} \right]. \end{aligned} \quad (\text{C.3})$$

C.2 Numerical results for $\bar{\Gamma}_{Z \rightarrow \bar{\nu}\nu}$

The δ correction to $\bar{\Gamma}_{Z \rightarrow \bar{\nu}\nu}$ is given by

$$\frac{\delta \bar{\Gamma}_{Z \rightarrow \bar{\nu}\nu}}{10^{-2}} = \left[-2.01 C_{H\ell}^{(1)} - 0.503 C_{HD} + 1.01 C_{\ell\ell} \right]. \quad (\text{C.4})$$

The $\delta \Delta$ correction to $\bar{\Gamma}_{Z \rightarrow \bar{\nu}\nu}$ has the contributions

$$\begin{aligned} \frac{\delta \Delta \bar{\Gamma}_{Z \rightarrow \bar{\nu}\nu}}{10^{-3}} = & \left[- (0.082 \Delta \bar{v}_T - 0.136) C_{H\ell}^{(1)} - (0.041 \Delta \bar{v}_T + 0.051) C_{HD} + 0.189 C_{\ell q}^{(3)} + 1.35 \Delta \bar{v}_T \right. \\ & \left. - (0.327 \Delta \bar{v}_T + 0.411) C_{H\ell}^{(3)} - (0.016 \Delta \bar{v}_T + 0.044) C_{HWB} + (0.204 \Delta \bar{v}_T + 0.138) C_{\ell\ell} \right], \end{aligned} \quad (\text{C.5})$$

and the $\delta \Delta$ corrections to $\bar{\Gamma}_{Z \rightarrow \bar{\nu}\nu}$ also has the logarithmic terms

$$\begin{aligned} \frac{\delta \Delta \bar{\Gamma}_{Z \rightarrow \bar{\nu}\nu}}{10^{-3}} = & \left[0.189 C_{H\ell}^{(1)} - 0.095 C_{HD} - 0.378 C_{H\ell}^{(3)} - 0.074 C_{HWB} + 0.284 C_{\ell\ell}, \right. \\ & + 0.378 (C_{\ell q}^{(1)} - C_{\ell u}) - 0.062 C_{uW} - 0.037 C_{uB} \left. \right] \log \left[\frac{\Lambda^2}{\hat{m}_t^2} \right], \\ & + \left[\left(5.93 C_{H\ell}^{(3)} + 7.41 C_{HD} \right) \times 10^{-6} + 0.033 C_{H\ell}^{(1)} - 0.003 C_{HWB} - 0.016 C_{\ell\ell} \right] \log \left[\frac{\Lambda^2}{\hat{m}_h^2} \right]. \end{aligned} \quad (\text{C.6})$$

C.3 Numerical results for $\bar{\Gamma}_{Z \rightarrow \bar{u}u}$

Similarly the δ correction to $\bar{\Gamma}_{Z \rightarrow \bar{u}u}$ is given by

$$\frac{\delta \bar{\Gamma}_{Z \rightarrow \bar{u}u}}{10^{-2}} = -1.37 C_{HD} - 5.49 C_{H\ell}^{(3)} - 4.16 \left(C_{Hq}^{(1)} - C_{Hq}^{(3)} \right) + 1.88 C_{Hu} - 0.559 C_{HWB} + 2.74 C_{\ell\ell}. \quad (C.7)$$

The $\delta \Delta$ correction to $\bar{\Gamma}_{Z \rightarrow \bar{u}u}$ has the contributions

$$\begin{aligned} \frac{\delta \Delta \bar{\Gamma}_{Z \rightarrow \bar{u}u}}{10^{-3}} = & \left[- (0.168 \Delta \bar{v}_T + 0.211) C_{HD} - (1.59 \Delta \bar{v}_T + 1.35) C_{H\ell}^{(3)} - (0.143 \Delta \bar{v}_T + 0.402) C_{Hu} \right. \\ & - (0.388 \Delta \bar{v}_T - 0.005) (C_{Hq}^{(1)} - C_{Hq}^{(3)}) - (0.093 \Delta \bar{v}_T + 0.138) C_{HWB} + 3.68 \Delta \bar{v}_T, \\ & \left. + 0.516 C_{\ell q}^{(3)} + (0.794 \Delta \bar{v}_T + 0.674) C_{\ell\ell} + 0.079 (C_{HB} + C_{HW}) \right], \end{aligned} \quad (C.8)$$

and the $\delta \Delta$ correction to $\bar{\Gamma}_{Z \rightarrow \bar{u}u}$ also has the logarithmic terms

$$\begin{aligned} \frac{\delta \Delta \bar{\Gamma}_{Z \rightarrow \bar{u}u}}{10^{-3}} = & \left[-0.389 C_{HD} - 1.62 C_{H\ell}^{(3)} - 0.117 \left(C_{Hq}^{(1)} - C_{Hq}^{(3)} \right) - 0.684 C_{Hu} - 0.248 C_{HWB}, \right. \\ & + 1.32 C_{\ell\ell} - 1.03 C_{\ell q}^{(3)} + 1.56 \left(C_{qq}^{(3)} + C_{qq}^{(1)} \right) - 1.13 C_{qu}^{(1)} + 0.706 C_{uu}, \\ & - 0.056 C_{uW} - 0.034 C_{uB} \left] \log \left[\frac{\Lambda^2}{\hat{m}_t^2} \right] + \left[2.31 \times 10^{-5} C_{HD} - 0.03 C_{Hu}, \right. \\ & \left. + 0.068 \left(C_{Hq}^{(1)} - C_{Hq}^{(3)} \right) - 0.003 C_{HWB} + 0.090 C_{H\ell}^{(3)} - 0.045 C_{\ell\ell} \right] \log \left[\frac{\Lambda^2}{\hat{m}_h^2} \right]. \end{aligned} \quad (C.9)$$

C.4 Numerical results for $\bar{\Gamma}_{Z \rightarrow \bar{d}d}$

The δ correction to $\bar{\Gamma}_{Z \rightarrow \bar{d}d}$ (where $d = \{d, s, b\}$) is given by

$$\frac{\delta \bar{\Gamma}_{Z \rightarrow \bar{d}d}}{10^{-2}} = -0.939 C_{Hd} - 1.58 C_{HD} - 6.31 C_{H\ell}^{(3)} + 5.10 \left(C_{Hq}^{(1)} + C_{Hq}^{(3)} \right) - 0.510 C_{HWB} + 3.15 C_{\ell\ell}. \quad (C.10)$$

The $\delta \Delta$ correction to $\bar{\Gamma}_{Z \rightarrow \bar{d}d}$ (where $d = \{d, s\}$) has the contributions

$$\begin{aligned} \frac{\delta \Delta \bar{\Gamma}_{Z \rightarrow \bar{d}d}}{10^{-3}} = & \left[(0.071 \Delta \bar{v}_T + 0.201) C_{Hd} - (0.115 \Delta \bar{v}_T + 0.144) C_{HD}, - (1.45 \Delta \bar{v}_T + 1.08) C_{H\ell}^{(3)} \right. \\ & + (0.316 \Delta \bar{v}_T - 0.206) \left(C_{Hq}^{(1)} + C_{Hq}^{(3)} \right) - (0.024 \Delta \bar{v}_T + 0.064) C_{HWB} + 4.23 \Delta \bar{v}_T, \\ & \left. + (0.727 \Delta \bar{v}_T + 0.541) C_{\ell\ell} + 0.593 C_{\ell q}^{(3)} + 0.072 (C_{HB} + C_{HW}) \right], \end{aligned} \quad (C.11)$$

and the $\delta \Delta$ corrections to $\bar{\Gamma}_{Z \rightarrow \bar{d}d}$ (where $d = \{d, s\}$) also has the logarithmic terms

$$\begin{aligned} \frac{\delta \Delta \bar{\Gamma}_{Z \rightarrow \bar{d}d}}{10^{-3}} = & \left[0.342 C_{Hd} - 0.266 C_{HD} - 0.995 C_{H\ell}^{(3)} - 0.225 \left(C_{Hq}^{(1)} + C_{Hq}^{(3)} \right) - 0.110 C_{HWB}, \right. \\ & + 1.09 C_{\ell\ell} - 1.19 C_{\ell q}^{(3)} + 0.176 \left(C_{qd}^{(1)} - C_{ud}^{(1)} \right) + 1.92 \left(C_{qq}^{(3)} - C_{qq}^{(1)} \right) + 0.958 C_{qu}^{(1)}, \\ & - 0.091 C_{uW} - 0.055 C_{uB} \left] \log \left[\frac{\Lambda^2}{\hat{m}_t^2} \right] + \left[(2.43 \times 10^{-5} C_{HD} + 0.015 C_{Hd}, \right. \\ & \left. + 0.103 C_{H\ell}^{(3)} - 0.083 \left(C_{hq}^{(1)} + C_{hq}^{(3)} \right) - 0.005 C_{HWB} - 0.052 C_{\ell\ell} \right] \log \left[\frac{\Lambda^2}{\hat{m}_h^2} \right]. \end{aligned} \quad (C.12)$$

C.5 Numerical results for $\bar{\Gamma}_{Z \rightarrow \bar{b}b}$

The δ correction to $\bar{\Gamma}_{Z \rightarrow \bar{b}b}$ is identical to that for other down-type quarks given in the previous subsection. The $\delta \Delta$ correction to $\bar{\Gamma}_{Z \rightarrow \bar{b}b}$ has the contributions

$$\begin{aligned} \frac{\delta \Delta \bar{\Gamma}_{Z \rightarrow \bar{b}b}}{10^{-3}} = & \left[(0.071 \Delta \bar{v}_T + 0.201) C_{Hd} - (0.115 \Delta \bar{v}_T + 0.221) C_{HD} - (1.45 \Delta \bar{v}_T + 1.59) C_{H\ell}^{(3)} \right. \\ & + (0.316 \Delta \bar{v}_T + 0.353) C_{Hq}^{(1)} + (0.316 \Delta \bar{v}_T + 0.016) C_{Hq}^{(3)} + 0.080 C_{Hu}, \\ & - (0.024 \Delta \bar{v}_T + 0.102) C_{HWB} + (0.727 \Delta \bar{v}_T + 0.796) C_{\ell\ell} + 0.593 C_{\ell q}^{(3)}, \\ & \left. + 1.28 C_{qq}^{(3)} + 0.101 C_{uH} + 0.072 (C_{HB} + C_{HW}) + 4.23 \Delta \bar{v}_T \right], \end{aligned} \quad (C.13)$$

and the $\delta \Delta$ corrections to $\bar{\Gamma}_{Z \rightarrow \bar{b}b}$ also has the logarithmic terms

$$\begin{aligned} \frac{\delta \Delta \bar{\Gamma}_{Z \rightarrow \bar{b}b}}{10^{-3}} = & \left[0.342 C_{Hd} - 0.317 C_{HD} - 1.33 C_{H\ell}^{(3)} - 0.066 C_{Hq}^{(1)} + 0.807 C_{Hq}^{(3)} - 0.160 C_{Hu}, \right. \\ & - 1.92 C_{qq}^{(1)} - 0.135 C_{HWB} + 1.26 C_{\ell\ell} - 1.19 C_{\ell q}^{(3)} + 0.176 (C_{qd}^{(1)} - C_{ud}^{(1)}), \\ & + 0.639 C_{qq}^{(3)} + 0.958 C_{qu}^{(1)} + 0.067 C_{uH} - 0.091 C_{uW} - 0.055 C_{uB} \left. \right] \log \left[\frac{\Lambda^2}{\hat{m}_t^2} \right], \\ & + \left[0.015 C_{Hd} + 2.43 \times 10^{-5} C_{HD} - 0.083 (C_{Hq}^{(1)} + C_{Hq}^{(3)}) + 0.103 C_{H\ell}^{(3)} - 0.005 C_{HWB} \right. \\ & \left. - 0.052 C_{\ell\ell} \right] \log \left[\frac{\Lambda^2}{\hat{m}_h^2} \right]. \end{aligned} \quad (C.14)$$

C.6 Numerical results for $\bar{\Gamma}_{Z \rightarrow \text{Had}}$

The δ correction to $\bar{\Gamma}_{Z \rightarrow \text{Had}}$ is given by

$$\begin{aligned} \frac{\delta \bar{\Gamma}_{Z \rightarrow \text{Had}}}{10^{-2}} = & \left[-2.82 C_{Hd} - 7.47 C_{HD} - 29.9 C_{H\ell}^{(3)} + 6.97 C_{Hq}^{(1)} + 23.6 C_{Hq}^{(3)} + 3.75 C_{Hu} \right. \\ & \left. - 2.65 C_{HWB} + 14.9 C_{\ell\ell} \right]. \end{aligned} \quad (C.15)$$

The $\delta \Delta$ correction to $\bar{\Gamma}_{Z \rightarrow \text{Had}}$ has the contributions

$$\begin{aligned} \frac{\delta \Delta \bar{\Gamma}_{Z \rightarrow \text{Had}}}{10^{-3}} = & \left[(0.214 \Delta \bar{v}_T + 0.603) C_{Hd} - (0.681 \Delta \bar{v}_T + 0.932) C_{HD} - (7.54 \Delta \bar{v}_T + 6.45) C_{H\ell}^{(3)}, \right. \\ & + (0.174 \Delta \bar{v}_T - 0.049) C_{Hq}^{(1)} + (1.73 \Delta \bar{v}_T - 0.406) C_{Hq}^{(3)} - (0.286 \Delta \bar{v}_T + 0.725) C_{Hu}, \\ & - (0.256 \Delta \bar{v}_T + 0.507) C_{HWB} + (3.77 \Delta \bar{v}_T + 3.22) C_{\ell\ell} + 1.28 C_{\ell q}^{(3)} + 20.0 \Delta \bar{v}_T, \\ & \left. + 2.81 C_{\ell q}^{(3)} + 0.101 C_{uH} + 0.374 (C_{HB} + C_{HW}) \right], \end{aligned} \quad (C.16)$$

and the logarithmic terms

$$\begin{aligned} \frac{\delta \Delta \bar{\Gamma}_{Z \rightarrow \text{Had}}}{10^{-3}} = & \left[1.03 C_{Hd} - 1.63 C_{HD} - 6.56 C_{H\ell}^{(3)} - 0.750 C_{Hq}^{(1)} + 0.590 C_{Hq}^{(3)} - 1.53 C_{Hu}, \right. \\ & - 0.85 C_{HWB} + 6.09 C_{\ell\ell} - 5.62 C_{\ell q}^{(3)} + 0.529 (C_{qd}^{(1)} - C_{ud}^{(1)}) + 7.60 C_{qq}^{(3)} + 0.605 C_{qu}^{(1)}, \\ & - 2.62 C_{qq}^{(1)} + 0.067 C_{uH} + 1.41 C_{uu} - 0.386 C_{uW} - 0.232 C_{uB} \left. \right] \log \left[\frac{\Lambda^2}{\hat{m}_t^2} \right], \\ & + \left[0.046 C_{Hd} + 1.19 \times 10^{-4} C_{HD} - 0.114 C_{Hq}^{(1)} - 0.386 C_{Hq}^{(3)} - 0.061 C_{Hu} + 0.489 C_{H\ell}^{(3)}, \right. \\ & \left. - 0.020 C_{HWB} - 0.244 C_{\ell\ell} \right] \log \left[\frac{\Lambda^2}{\hat{m}_h^2} \right]. \end{aligned} \quad (C.17)$$

C.7 Numerical results for $\bar{\Gamma}_Z$

The δ correction to $\bar{\Gamma}_Z$ is given by

$$\begin{aligned} \frac{\delta \bar{\Gamma}_Z}{10^{-2}} = & \left[-2.82 \left(C_{Hd} + C_{He} + C_{H\ell}^{(1)} \right) - 9.87 C_{HD} - 30.2 C_{H\ell}^{(3)} + 6.97 C_{Hq}^{(1)} + 23.6 C_{Hq}^{(3)} \right. \\ & \left. + 3.75 C_{Hu} - 2.80 C_{HWB} + 19.7 C_{\ell\ell} \right]. \end{aligned} \quad (C.18)$$

Similarly, the $\delta \Delta$ correction to $\bar{\Gamma}_Z$ has the contributions

$$\begin{aligned} \frac{\delta \Delta \bar{\Gamma}_Z}{10^{-3}} = & \left[(0.214 \Delta \bar{v}_T + 0.603) \left(C_{Hd} + C_{He} + C_{H\ell}^{(1)} \right) - (1.09 \Delta \bar{v}_T + 1.44) C_{HD}, \right. \\ & - (9.69 \Delta \bar{v}_T + 9.11) C_{H\ell}^{(3)} + (0.174 \Delta \bar{v}_T - 0.049) C_{Hq}^{(1)} + (1.73 \Delta \bar{v}_T - 0.406) C_{Hq}^{(3)}, \\ & - (0.286 \Delta \bar{v}_T + 0.725) C_{Hu} - (0.560 \Delta \bar{v}_T + 1.00) C_{HWB}, \\ & + (5.20 \Delta \bar{v}_T + 4.45) C_{\ell\ell} + 3.71 C_{\ell q}^{(3)} + 1.28 C_{qq}^{(3)}, \\ & \left. + 0.101 C_{uH} + 0.395 (C_{HB} + C_{HW}) + 26.5 \Delta \bar{v}_T \right], \end{aligned} \quad (C.19)$$

and the $\delta \Delta$ correction to $\bar{\Gamma}_Z$ also has the logarithmic terms

$$\begin{aligned} \frac{\delta \Delta \bar{\Gamma}_Z}{10^{-3}} = & \left[1.03 \left(C_{Hd} + C_{He} + C_{H\ell}^{(1)} \right) - 2.56 C_{HD} - 9.66 C_{H\ell}^{(3)} - 0.749 C_{Hq}^{(1)} + 0.590 C_{Hq}^{(3)}, \right. \\ & - 1.53 C_{Hu} - 1.71 C_{HWB} + 8.49 C_{\ell\ell} - 5.69 C_{\ell q}^{(3)} + 7.60 C_{qq}^{(3)}, \\ & + 0.529 \left(C_{\ell q}^{(1)} + C_{qd}^{(1)} + C_{qe} + C_{qd}^{(1)} - C_{\ell u} - C_{ud}^{(1)} - C_{eu} \right) \\ & - 2.62 C_{qq}^{(1)} + 0.605 C_{qu}^{(1)} + 0.067 C_{uH} + 1.41 C_{uu} - 0.651 C_{uW} - 0.391 C_{uB} \Big] \log \left[\frac{\Lambda^2}{\hat{m}_t^2} \right], \\ & + \left[0.046 \left(C_{Hd} + C_{He} + C_{H\ell}^{(1)} \right) + 1.60 \times 10^{-4} C_{HD}, - 0.114 C_{Hq}^{(1)} - 0.386 C_{Hq}^{(3)}, \right. \\ & \left. - 0.061 C_{Hu} + 0.495 C_{H\ell}^{(3)} - 0.323 C_{\ell\ell} - 0.034 C_{HWB} \right] \log \left[\frac{\Lambda^2}{\hat{m}_h^2} \right]. \end{aligned} \quad (C.20)$$

C.8 Numerical results for \bar{R}_ℓ^0

The δ correction to \bar{R}_ℓ^0 is given by

$$\begin{aligned} \frac{\delta \bar{R}_\ell^0}{10^{-2}} = & -33.8 C_{Hd} + 226 C_{He} - 258 C_{H\ell}^{(1)} - 18.2 C_{HD} - 331 C_{H\ell}^{(3)} + 83.7 C_{Hq}^{(1)} \\ & + 283 C_{Hq}^{(3)} + 45.1 C_{Hu} - 19.9 C_{HWB} + 36.3 C_{\ell\ell}. \end{aligned} \quad (C.21)$$

Similarly, the $\delta \Delta$ correction to \bar{R}_ℓ^0 has the contributions

$$\begin{aligned} \frac{\delta \Delta \bar{R}_\ell^0}{10^{-3}} = & \left[- (33.3 \Delta \bar{v}_T + 66.4) C_{He} - (18.4 \Delta \bar{v}_T - 4.85) C_{H\ell}^{(1)} + (5.81 \Delta \bar{v}_T + 11.3) C_{Hd}, \right. \\ & + (17.9 \Delta \bar{v}_T + 22.3) C_{HD} + (35.6 \Delta \bar{v}_T + 78.1) C_{H\ell}^{(3)} - (5.93 \Delta \bar{v}_T + 10.7) C_{Hq}^{(1)}, \\ & - (6.39 \Delta \bar{v}_T + 40) C_{Hq}^{(3)} - (7.74 \Delta \bar{v}_T + 14.1) C_{Hu} + (19.6 \Delta \bar{v}_T + 24.9) C_{HWB}, \\ & \left. - (27.0 \Delta \bar{v}_T + 36.6) C_{\ell\ell} + 6.83 C_{\ell q}^{(3)} + 15.3 C_{qq}^{(3)} + 1.21 C_{uH} + 2.81 (C_{HB} + C_{HW}) + 48.6 \Delta v \right], \end{aligned} \quad (C.22)$$

and the $\delta \Delta$ correction to \bar{R}_ℓ^0 also has the logarithmic terms

$$\begin{aligned} \frac{\delta \Delta \bar{R}_\ell^0}{10^{-3}} = & \left[19.8 C_{Hd} - 118 C_{He} + 4.18 C_{H\ell}^{(1)} + 41.3 C_{HD} + 154 C_{H\ell}^{(3)} - 27.5 C_{Hq}^{(1)} - 55.7 C_{Hq}^{(3)}, \right. \\ & - 28.3 C_{Hu} + 46.0 C_{HWB} - 68.2 C_{\ell\ell} + 48.5 (C_{\ell q}^{(1)} - C_{\ell u}) - 62.2 C_{\ell q}^{(3)} + 6.36 C_{qd}^{(1)} \\ & - 42.4 (C_{qe} - C_{eu}) - 31.5 C_{qq}^{(1)} + 91.2 C_{qq}^{(3)} + 7.26 C_{qu}^{(1)} \\ & + 0.810 C_{uH} - 6.36 C_{ud}^{(1)} + 16.9 C_{uu} + 1.78 C_{uW} + 1.07 C_{uB} \Big] \log \left[\frac{\Lambda^2}{\hat{m}_t^2} \right], \\ & + \left[0.552 C_{Hd} - 1.48 \times 10^{-4} C_{HD} - 3.69 C_{He} + 4.22 C_{H\ell}^{(1)} + 5.40 C_{H\ell}^{(3)}, \right. \\ & \left. - 1.37 C_{Hq}^{(1)} - 4.63 C_{Hq}^{(3)} - 0.736 C_{Hu} - 0.593 C_{\ell\ell} + 0.092 C_{HWB} \right] \log \left[\frac{\Lambda^2}{\hat{m}_h^2} \right]. \quad (C.23) \end{aligned}$$

C.9 Numerical results for \bar{R}_b^0

The δ correction to \bar{R}_ℓ^b is given by

$$\begin{aligned} \frac{\delta R_b^0}{10^{-2}} = & -0.192 C_{Hd} + 0.039 C_{HD} + 0.158 C_{H\ell}^{(3)} + 2.13 C_{Hq}^{(1)} - 0.055 C_{Hq}^{(3)}, \\ & -0.494 C_{Hu} + 0.043 C_{HWB} - 0.079 C_{\ell\ell}. \quad (C.24) \end{aligned}$$

Similarly, the $\delta \Delta$ correction to \bar{R}_b^0 has the contributions

$$\begin{aligned} \frac{\delta \Delta R_b^0}{10^{-3}} = & \left[(0.036 \Delta \bar{v}_T + 0.083) C_{Hd} + (0.011 \Delta \bar{v}_T + 0.013) C_{HD} + (0.084 \Delta \bar{v}_T - 0.014) C_{H\ell}^{(3)}, \right. \\ & - (0.085 \Delta \bar{v}_T + 0.152) C_{Hq}^{(1)} - (0.016 \Delta \bar{v}_T + 0.019) C_{Hq}^{(3)} + (0.099 \Delta \bar{v}_T + 0.208) C_{Hu}, \\ & - (0.042 \Delta \bar{v}_T - 0.007) C_{\ell\ell} + (0.013 \Delta \bar{v}_T + 0.009) C_{HWB} - 0.015 C_{\ell q}^{(3)}, \\ & \left. + 0.597 C_{qq}^{(3)} + 0.047 C_{uH} - 0.006 (C_{HB} + C_{HW}) - 0.106 \Delta v \right], \quad (C.25) \end{aligned}$$

and the $\delta \Delta$ correction to \bar{R}_b^0 also has the logarithmic terms

$$\begin{aligned} \frac{\delta \Delta R_b^0}{10^{-3}} = & \left[0.129 C_{Hd} + 0.025 C_{HD} + 0.067 C_{H\ell}^{(3)} - 0.559 C_{Hq}^{(1)} + 0.383 C_{Hq}^{(3)} + 0.240 C_{Hu}, \right. \\ & + 0.023 C_{HWB} - 0.049 C_{\ell\ell} + 0.030 C_{\ell q}^{(3)} + 0.036 (C_{qd}^{(1)} - C_{ud}^{(1)}) - 0.618 C_{qq}^{(3)}, \quad (C.26) \\ & - 0.803 C_{qq}^{(1)} + 0.494 C_{qu}^{(1)} - 0.002 C_{uB} + 0.032 C_{uH} - 0.004 C_{uW} - 0.186 C_{uu} \Big] \log \left[\frac{\Lambda^2}{\hat{m}_t^2} \right] \\ & + \left[-8.94 \times 10^{-7} C_{HD} + (0.313 C_{Hd} - 3.49 C_{Hq}^{(1)} + 0.090 C_{Hq}^{(3)} - 0.258 C_{H\ell}^{(3)}, \right. \\ & \left. + 0.808 C_{Hu} + 0.129 C_{\ell\ell} - 0.020 C_{HWB}) 10^{-2} \right] \log \left[\frac{\Lambda^2}{\hat{m}_h^2} \right]. \end{aligned}$$

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